AN INVESTIGATION OF SCIENTIFIC INVENTORY MANAGEMENT TECHNIQUES APPLICABLE TO THE NAVY EXCHANGE PROGRAM

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By

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B.S., United States Naval Academy, 1949

Submitted to the Department of Chemical and Petroleum Engineering and the Faculty of the Graduate School of The University of Kansas in Partial Fulfillment of the Requirements for the Degree of Master of Science

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ACKNOWLEDGEMENTS

The author wishes to express his sincere appreciation for the invaluable assistance provided so willingly by Dr. Floyd W. Preston, not only in regard to work on this thesis, but throughout the entire pursuit of his studies in the Petroleum Engineering Curriculum.

To Dr. Charles F. Weinaug, a word of thanks for his overall guidance in the Petroleum Management Program.

To Dean Wiley S. Mitchell, a special word of appreciation is due for providing the author with a fascinating insight of Management Accounting, and helpful counsel in pursuing his studies in Business Administration.

The author is greatly indebted to the United States Navy for granting him the priceless opportunity of continuing his education at the graduate level.

Finally, the author is deeply grateful to his wife, Virginia, for her continual encouragement and assistance throughout the past fourteen months.

TABLE OF CONTENTS

TABLE OF CONTENTS (Continued)

LIST OF TABLES

LIST OF TABLES (Continued)

LIST OF FIGURES

CHAPTER I

INTRODUCTION

Background

Inventory - "Goods or stock; an itemized list of goods with their estimated worth."[1] To the farmer, crops in the fields represent inventory. In broader terms, inventory connotes not only the finished goods awaiting shipment by a manufacturer, but also machines, machine parts, tools, personnel, trucks, cash and auxiliary equipment.[2] Within retailing organizations, the term "inventory" specifically applies to saleable merchandise located in the warehouse or on the salesfloor. Inventory, then, is a term that means many things to many people, but to the farmer, the manufacturer, and the retailer, it always equates to one common denominator - money. Efficient inventory management adds to profits; conversely, inefficient inventory management adds to expense, thereby reducing profits.

Every retail business carries an inventory which is quite dynamic. Its level changes daily, depending on the relationship of that day's sales to the receipt of goods. Depending on the type of business in which the retail firm is engaged, the material costs of inventory, i.e. the cost of goods sold, represent 50 to 90 percent of the total cost of doing business.[3] Because of the dynamic nature of a

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[1]References at end.

retailer's inventory, coupled with its prominence as a major total cost factor, a large measure of a firm's success will depend upon just how efficiently inventory is managed.

Many successful managers with a sensitive "feel" for the cyclic trends of their business have managed inventories on an intuitive basis. They know from long experience "when" and "how much" to order so as to maintain a relatively low inventory, while at the same time avoiding a "not in stock" position most of the time. However, "intuitive" management of today's giant retail operations carrying as many as 500,000 SKU's (stock keeping units)[4] of merchandise is just as much out of the question as it would be for a commercial pilot to fly a modern jet transport "by the seat of his pants." Though effective in past years, these holdovers of a bygone era of merchandising have found their effectiveness gradually diminishing as they competed with a new type of manager who understood and applied scientific management techniques.

The foregoing statement is not intended to imply that experience and sound judgement can be wholly supplanted by scientific methods. They cannot. Rather, there must be a logical blending of a veteran manager's knowledge gained through experience, with the new management techniques and devices available to him, to provide a system superior to that of either when taken separately. The judicious blending of experience with scientific methods of handling inventory furnish management with an extremely valuable and indispensable tool - scientific inventory control.

It is, indeed, unfortunate that the retail trade has been far less receptive toward adoption of the "formula approach" of doing business than their prime suppliers, the manufacturers. Cost control in the manufacturing industry became a recognized tool in realizing vast economies in mass production of consumer goods in the early 1900's.[5] As early as 1926, a few far-sighted experts were writing about inventory management formulas in trade journals.[6] Although adopted by a few equally far-sighted retail managers, the scientific approach which they expounded was largely ignored by those schooled in the traditional way of thinking that intelligent decisions involving inventories could be made solely on the basis of hard earned knowledge, gained through long apprenticeship in the retail trade.

It is a paradox that in time of war, a country's resources are wasted on a grandiose scale yet, at the same time, the pressures of wartime shortage bring about development of new theories and techniques which tend to offset this inherent waste. As a prime example, World War II produced a new scientific discipline known as "Operations research", a discipline which has for its purpose the optimization of resource use, or as stated by Jenny, "Operations research is optimizing the performance of a system. This requires the application of scientific methods, techniques and tools."[7]

In the post-war years, men trained in scientific disciplines infused both industry and the retail trade with their

newly perfected scientific techniques and tools. Perhaps the pressure brought about by the narrowing profit-expense gap, due in large measure to the entry of the Discount Store on the retailing scene, served as a much needed catalyst to bring about universal acceptance and adoption of scientific methods in retail operations. Retailers were forced to break with many traditional practices and become increasingly more efficient in order to prevent a steady decline of profits.

Since inventory costs make up such a large percentage of the total cost of doing business, inventory management has proven to be a fertile field in which to realize significant economies for the firm. As more "intuitive" managers have come to realize the benefits accruing from application of scientific methods, scientific inventory management has become solidly entrenched in the business practices of large retail operations. Today, with the advent of the electronic computer and compact, relatively inexpensive data processing systems, coupled with newly developed inventory control techniques for both machine and manual application, it is increasingly apparent that the inventory management problem can be solved by companies of virtually any size utilizing scientifically based techniques.[8]

What is Scientific Inventory Management?

In the preceding section, the term "scientific inventory management" was introduced along with "scientific

inventory control." Both terms are used interchangeably by various authors in the literature dealing with this important functional area.[9] Both refer to the application of scientific techniques in the solution of a variety of inventory management problems. In essence, these techniques involve quantification of decision parameters and routine utilization of basic mathematical formulas in arriving at the most economical solution to a given inventory problem.

As stated by Brown,[10] "An inventory-control system is a coordinated set of rules for answering routinely the questions of when and how much to order, and for calling attention to the non-routine situations that the rules do not cover."

<u>The Navy Exchange Program.</u>

In the rather prosaic wording of the Navy Exchange Manual, the mission of the Navy Exchange is as follows:[11]

> "To provide a convenient and reliable source from which authorized patrons may obtain, at the lowest practicable cost, merchandise and services required for their well-being and contentment;
>
> To provide through accrued profits a source of funds to be used to supplement funds appropriated for the welfare and recreation of Naval personnel;
>
> To promote the morale of the command in which established through the establishment of a well managed, attractive and highly serviceable store."

Therein lie the same basic reasons for establishing any civilian retail operation - the profit motive and an intent to provide desired merchandise and/or services. In concept,

the Navy Exchange Program, which is made up of 169 individual Exchanges, can be likened to a Sears and Roebuck or J. C. Penney operation which is tailored to serve the needs of the Navy Community, both in the United States and overseas.

During fiscal year 1964, aggregate sales volume of the Exchanges totalled $368,432,000.00, of which sales in the retail departments amounted to $210,211,000.00.[12] Service departments such as gas stations, restaurants, laundries, vending machines, etc. contributed the remainder of $158,221,000.00. The record volume of annual sales realized in fiscal year 1964 places the Exchange Program 32nd among all retail organizations in the country.[13] This information is provided for the sole purpose of viewing the operation of the Exchange Program in its proper perspective; that of a giant among the country's retail organizations. Inventory control methods, which have proven worth to large civilian retail organizations, must be equally beneficial when applied in the Navy Exchanges.

Scientific Inventory Management In The Navy Exchange Program.

Although service to its patrons is the prime consideration in a Navy Exchange, the need to produce a profit is only slightly less important.[14] Contrary to popular belief, Exchange operations are not subsidized through funds appropriated by Congress. Funds required for the financing of new facilities and remodelling of old, as well as working capital needs are drawn from the operation itself, without

recourse even to commercial bank loans. Although exigencies of the service may dictate that a particular department of the Exchange operate at a loss, the overall operation must produce a profit. Furthermore, it must produce a profit each month rather than simply on an annual basis. In order to fulfill its mission, therefore, each Navy Exchange must strive for maximum efficiency of operation. And the key to efficient inventory management lies in the routine implementation of scientific inventory management methods.

Purpose

The purpose of this thesis is two-fold. First, the scientific basis of inventory management decision-making is investigated through the use of mathematical models. In particular, the traditional questions posed in any retail operation of "When do I order?", and "How much do I order?" will be answered by means of a thorough analysis of the Reorder Point formula and the Economic Order Quantity (EOQ) Concept. Second, an extremely useful but little known device for inventory managers called the Optimal Policy Curve will be developed, and its applicability to the Navy Exchange Program investigated.

General Discussion

The terms "economic order quantity" and "optimal order quantity" will be used extensively throughout subsequent chapters. Although these terms would appear to have the same

connotation, in fact they do not. The difference goes beyond mere semantics. The EOQ order policy has but one objective - to optimize, i.e. minimize, the total variable cost of inventory management.[15] In accomplishing this objective, other important factors such as availability of adequate warehouse space and staffing requirements of the purchase functions are ignored. Merchandise is ordered in quantities dictated by the EOQ formula. The order quantity is solely a function of order cost, inventory carrying cost, and annual item sales, and does not operate within any limits of practical constraints in regard to acceptable inventory levels or total orders processed annually. Expressed mathematically:

$$Q = f(C_o, I, S).$$

The penalty paid for cost optimization through application of the EOQ Concept is loss of flexibility in a firm's order policy.

On the other hand, the goal of the optimal order policy is to minimize total variable inventory management costs <u>under imposed restraints</u> of working inventory level and/or total number of orders the firm is capable of processing annually.[16] In a typical situation in which an Exchange is operating on a monthly order cycle, adoption of an EOQ order policy would undoubtedly result in total variable cost reductions, but probably at the expense of an increase in inventory level. If the Exchange is hampered by inadequate storage space to begin with, the expected savings imputed

to an EOQ order policy would be offset by increased deterioration of merchandise through damage, higher shrinkage rates, etc. These factors are not taken into consideration in a straightforward application of the EOQ formula.

The optimal policy seeks the optimum relationship between working inventory level and numbers of orders processed annually within practical limits imposed by the reality of the situation.[17] The reader must understand the underlying significance of the difference between these two order policies, and the terms used in describing same, if he is to derive any benefit from the treatment accorded them in Chapters III and IV.

The material contained within this thesis will hopefully serve to bridge the gap between operating instructions found in the Navy Exchange Stock Control Handbook and the Navy Exchange Manual, and the basic mathematical models and formulas from which these instructions evolve. In order to be understood and properly carried out at the working level, instructions governing the application of scientific inventory control procedures must be simple and precise. The stock clerk who must determine daily the order quantity of a hundred or more SKU's cannot, and will not, concern himself with the subtleties of the EOQ concept, nor with the statistical reorder point. It is sufficient that he carry out by rote the step-by-step procedures to be followed, as for example those outlined on page II-17 of the stock control

handbook[18] - to wit; when the stock balance of item X drops to level Y (reorder point), procurement action must be initiated for Z units (order factor table quantity) of item X.

In all probability, the average stock clerk will follow the handbook procedures mechanically with an admirably low incidence of errors and a not so admirable, and even lower, level of comprehension of the significance of his actions. Accuracy in making hundreds of daily postings to stock cards and in carrying out simple additions and subtractions is the prime requisite of an efficient stock clerk. However, at the supervisory and managerial levels, it does not suffice to have total recall of each printed word in the handbook if such complete familiarization of stock control procedures is not accompanied by a reasonable understanding of the theory involved. Subsequent chapters of this thesis will explore the mathematical theory that provides the "why we do it" answer to the "what do we do?" question.

The scope of scientific techniques discussed in this thesis is limited to management of the staple item portion of a Navy Exchange inventory. These are the items which are warehoused, and for which there is a year-round demand. Although there is some over-lap in techniques employed, scientific inventory management methods applicable to the control of fashion goods, "fad" items, or merchandise of a seasonal nature differ considerably from the techniques investigated herein. They are sufficiently diverse to provide ample subject matter for a separate study.

In arriving at the choice of this particular thesis topic, the author attempted to direct his efforts toward a management functional area which, though explored in trade journals and management texts of recent years, has been glossed over too lightly by the vast majority of retail managers. Because of his close association with the Navy Exchange Program, the author is familiar with the daily problems encountered in inventory management. Like his civilian counterparts, he has made all too many decisions based upon an "intuitive feel for the problem" which came from considerable - and frequently painful - experience in a similar situation. How much better these decisions would have been, had the author been more familiar with the scientific techniques explored in this thesis, is a matter of conjecture. Let it suffice to say that he has learned much of value in his research on this subject which he hopes to pass on to others interested in the area of scientific inventory management. In particular, the author would hope that the subject matter of this thesis will be of some benefit to all Navy Exchange Managers, and to other Navy Supply Corps Officers, who, like himself, have a continuing interest, not only in the Navy Exchange Program, but in all management areas.

CHAPTER II

BASIC PRINCIPLES OF INVENTORY CONTROL

Maximize Effectiveness of Control Effort.

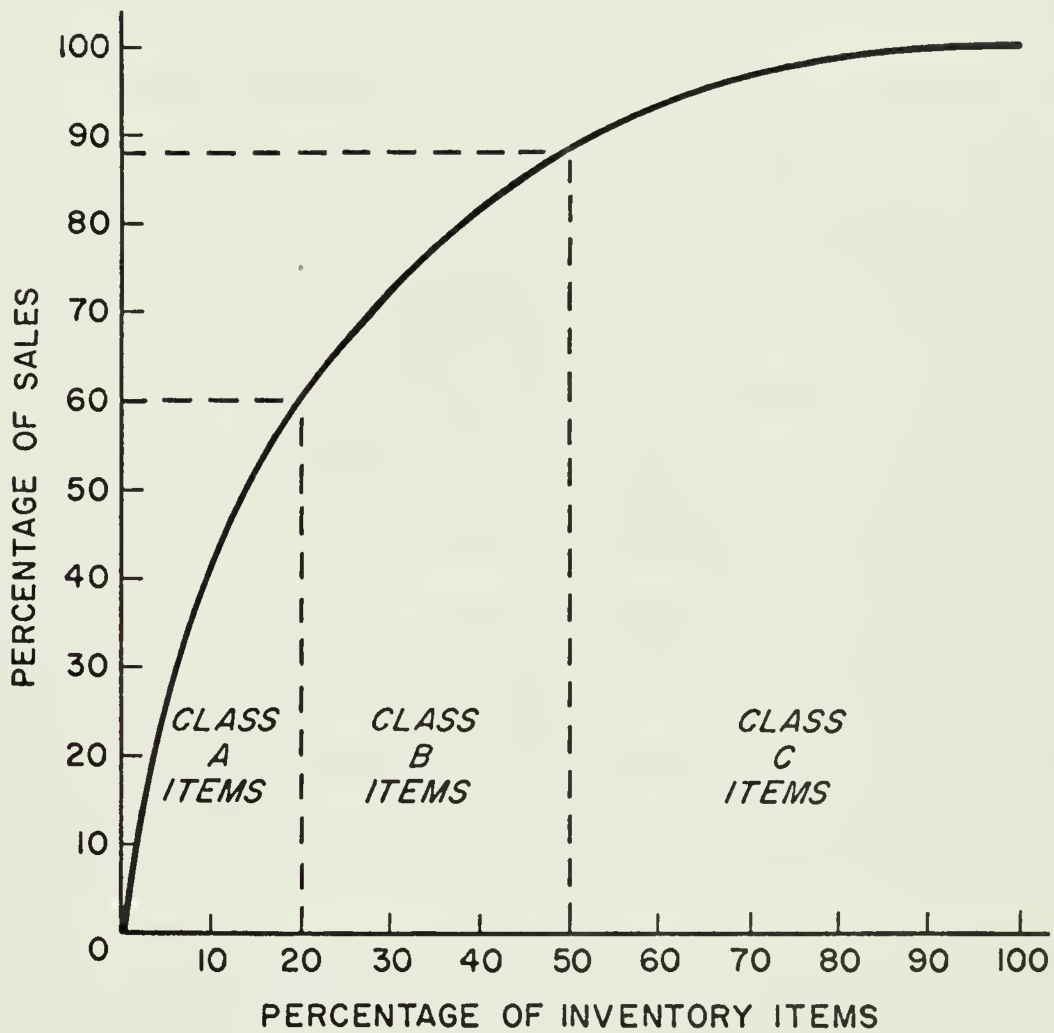
In a typical Group VI Navy Exchange[19], as many as 5,000 SKU's of merchandise are warehoused, and an additional 5,000 or more items are transferred directly to the salesfloor after receipt and pricing have been accomplished. It should be obvious, even to a person completely unfamiliar with the science of inventory control, that equal attention cannot be given to the management of each of these units of inventory. A year's sales of white 27 inch shoe laces might amount to less than $10.00, whereas Pall Mall cigarettes alone might generate sales in excess of $100,000.00 per annum.

Indeed, it has been determined that in a typical civilian retail operation, up to 80% of the annual dollar usage (sales) in a given inventory is concentrated in 25% of the items carried in stock[20]. A similar distribution pattern holds true within Navy Exchanges, as illustrated by the dollar sales - number of items carried relationship at the Navy Exchange, NAB Little Creek, Va., plotted in Figure II-1. Inventory items have been divided into three classes, A, B, and C. This is an adaptation to retailing of the "ABC inventory system" frequently encountered in manufacturing[21]. By utilizing this simple method of classification, an inventory control system can be designed to concentrate control

FIGURE II - I

Distribution of Inventory Items by Dollar Value of Sales

A B C CLASSIFICATION



effort on the area in which it will be most effective. The "must" and "never out" lists published for use in Navy Exchanges[22] are analogous to classes A and B respectively, while all other merchandise carried by Exchanges could be considered as falling into the category of class C items.

In an inventory such as that depicted in Figure II-1, an improvement in efficiency in managing the top 20% of dollar value items would produce a 50% greater cost savings than an equal improvement in efficiency in handling the remaining 80% of the stock. Maximum effort must therefore be applied to the high dollar usage merchandise in order to minimize inventory management costs. In short, the limited amount of available control effort must be allocated selectively so that funds expended on individual unit control will be in proportion to its importance. The greater the sales potential, profit contribution potential, service provided to the patron by the item, or inventory investment in the item, the greater the degree of management control effort allocated to it[23].

This concept of applying inventory management funds to those items which will produce the greatest return is basic to the successful operation of any business enterprise, yet it is frequently ignored by some managers who tend to give to each item of merchandise the same degree of attention.

The Dual Problem of Stock-out And Overstock.

Every retail manager has been confronted on countless occasions with the twin nightmares of stockouts in some items,

and a glut in others. Although a perfect remedy for these two serious inventory problems will never be attained, an effective inventory control system will greatly reduce their frequency of occurrence. Because of their deleterious effect on service and sales, stock-outs normally create the more serious problems. However, extreme cases of across-the-board overstocking can lead to disastrous financial problems if not remedied in time.

The Inventory Cycle.

Every inventory model is made up of three basic functions; procurement, storage, and issue (usage). Figure II-2 illustrates an inventory cycle with constant demand (sales) and leadtime, in which the purchase quantity "q" is received just as the last item of inventory has been issued or sold. The inventory level at time, $t = 0$ is therefore q. Issues are made from inventory at a constant rate to meet demand, represented by the slope of the line Y_1X_2. When the level of inventory drops to a pre-determined point R (reorder point), the quantity q is ordered from the supplier. The stock on hand at the reorder point is calculated to cover usage requirements during lead time and the time t_1 represents the leadtime for the particular item being ordered. Usage continues at a uniform rate until at $t = t$, the supply is depleted, the shelf is empty, and the stock card records a zero balance. At that instant, the purchased quantity arrives. Once again, inventory jumps to a level of q units (or dollars) and the cycle is repeated.

FIGURE II-2

An Inventory Cycle Illustrating Constant Demand and Leadtime

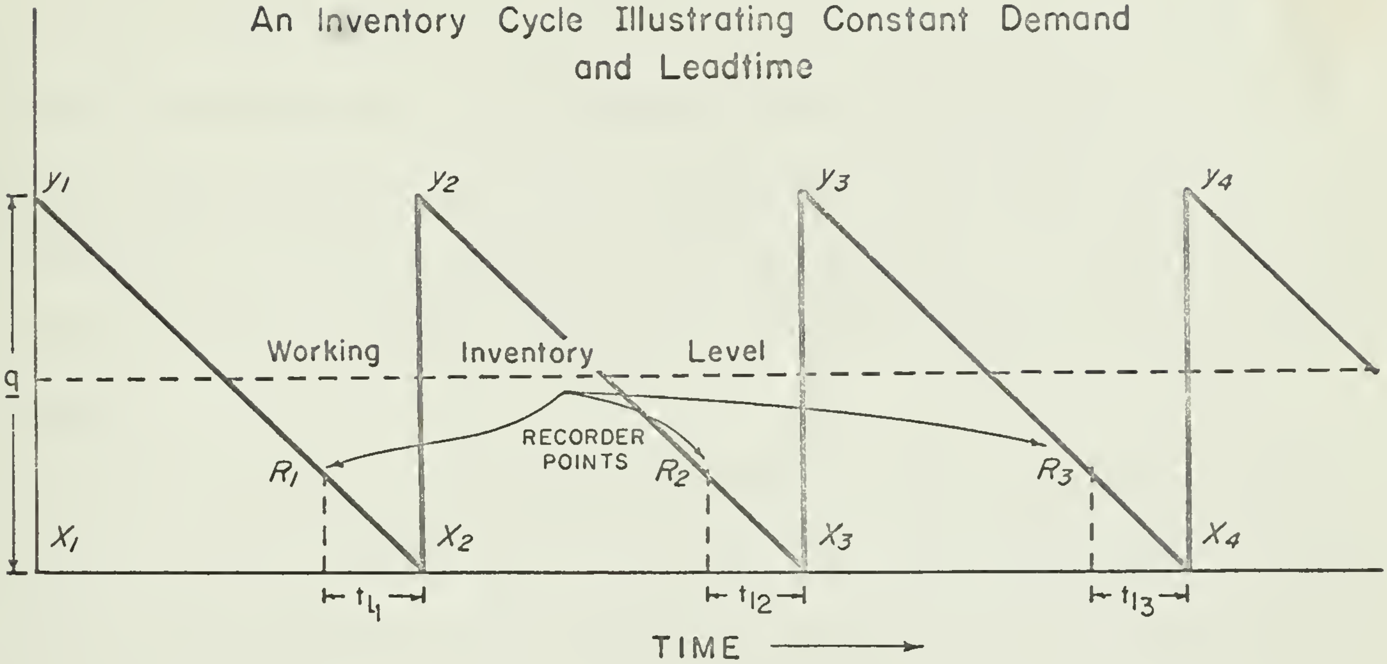
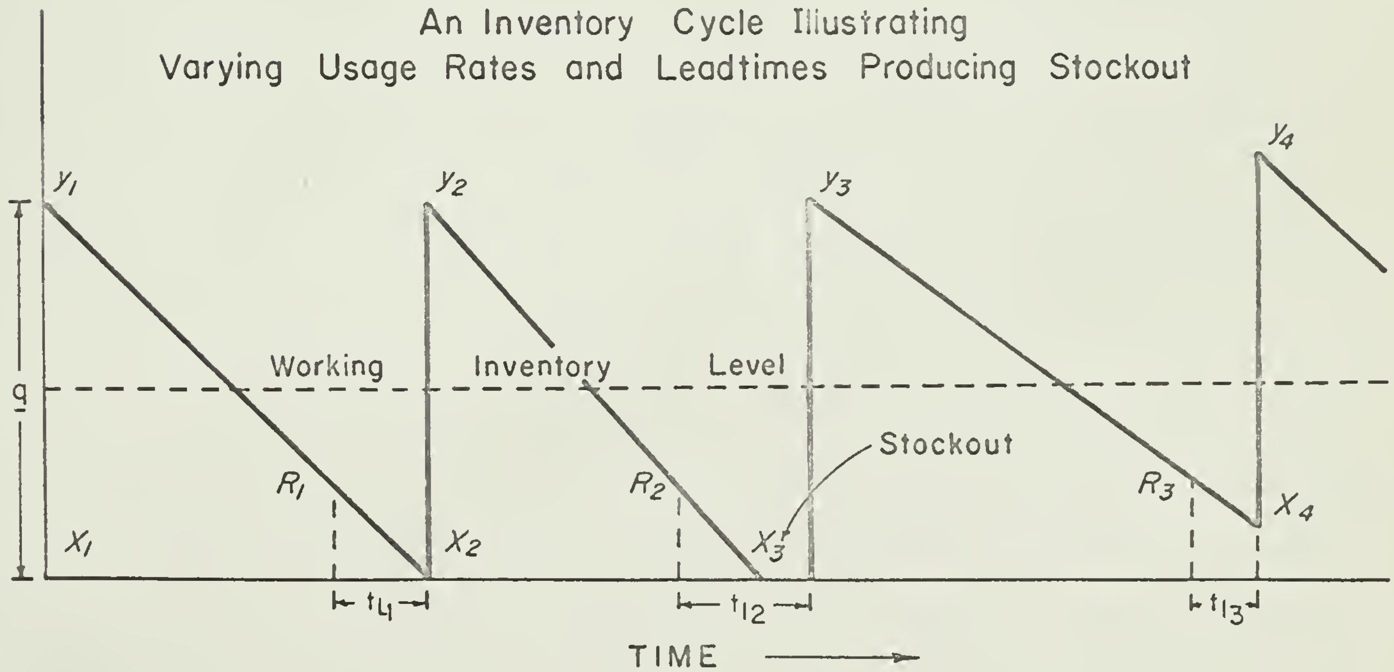


FIGURE II-3

An Inventory Cycle Illustrating Varying Usage Rates and Leadtimes Producing Stockout



The steady state cycle depicted in Figure II-2 does not accurately represent the dynamics of a real business inventory. However, its study is useful in illustrating the basic mechanics of an inventory cycle. The quantity (q/2), called the working inventory, appears frequently in formulas discussed in later sections of this thesis. It differs from the average inventory level by an amount equal to the safety stock.

Since the idealized behavior of the inventory model portrayed in Figure II-2 does not occur in practice, a somewhat more sophisticated approach is taken in the model shown in Figure II-3. Usage rates, indicated by the slopes of the lines $Y_1 X_2$, $Y_2 X_3$, and Y_3X_4 vary, as do the leadtimes $t_{\ell 1}$, $t_{\ell 2}$, and $t_{\ell 3}$. During the second time period, an increase in usage rate coupled with a longer than expected lead time has produced a stockout.

In any retail business, civilian or military, there is considerable variation in demand. This is particularly true in Navy Exchanges serving fleet units or Air Squadrons which deploy on fleet exercises several times a year, and often on very short notice. Some variation is also experienced in leadtime although for the most part it is relatively small. As previously explained, variations in either usage or leadtime, or both, can lead to stockouts. Since stockouts are a prime source of lost sales and patron dissatisfaction, they must be rigorously avoided. The obvious solution to the problem lies in adding a cushion of extra stock, called safety stock, or buffer stock, to the working inventory.

In Figure II-4, varying leadtimes are combined with widely fluctuating usage. However, the addition of a safety stock has effectively prevented stockouts. The inventory model depicted in Figure II-4, though somewhat simplified, graphically illustrates the inventory cycle in a typical Exchange for one SKU of merchandise. Its salient features are the safety stock, average inventory, working inventory, order quantity, reorder point, leadtimes and usage rates. A useful mental classification of these factors is to consider the first five as dependent variables, whereas leadtime and usage rates are controlling or independent variables.

Working Inventory.

The average of a variable, INV, which can be considered to be dependent upon time t, is obtainable for any form of the function from the relation

$$INV_{avg.} = \frac{\int_{T_1}^{T_2} INV\ dt}{T_2 - T_1} \qquad \text{(II-1)}$$

This relation corresponds to the definition of the "integrated average" of a function. If we assume that the variable INV is a linear function of time,

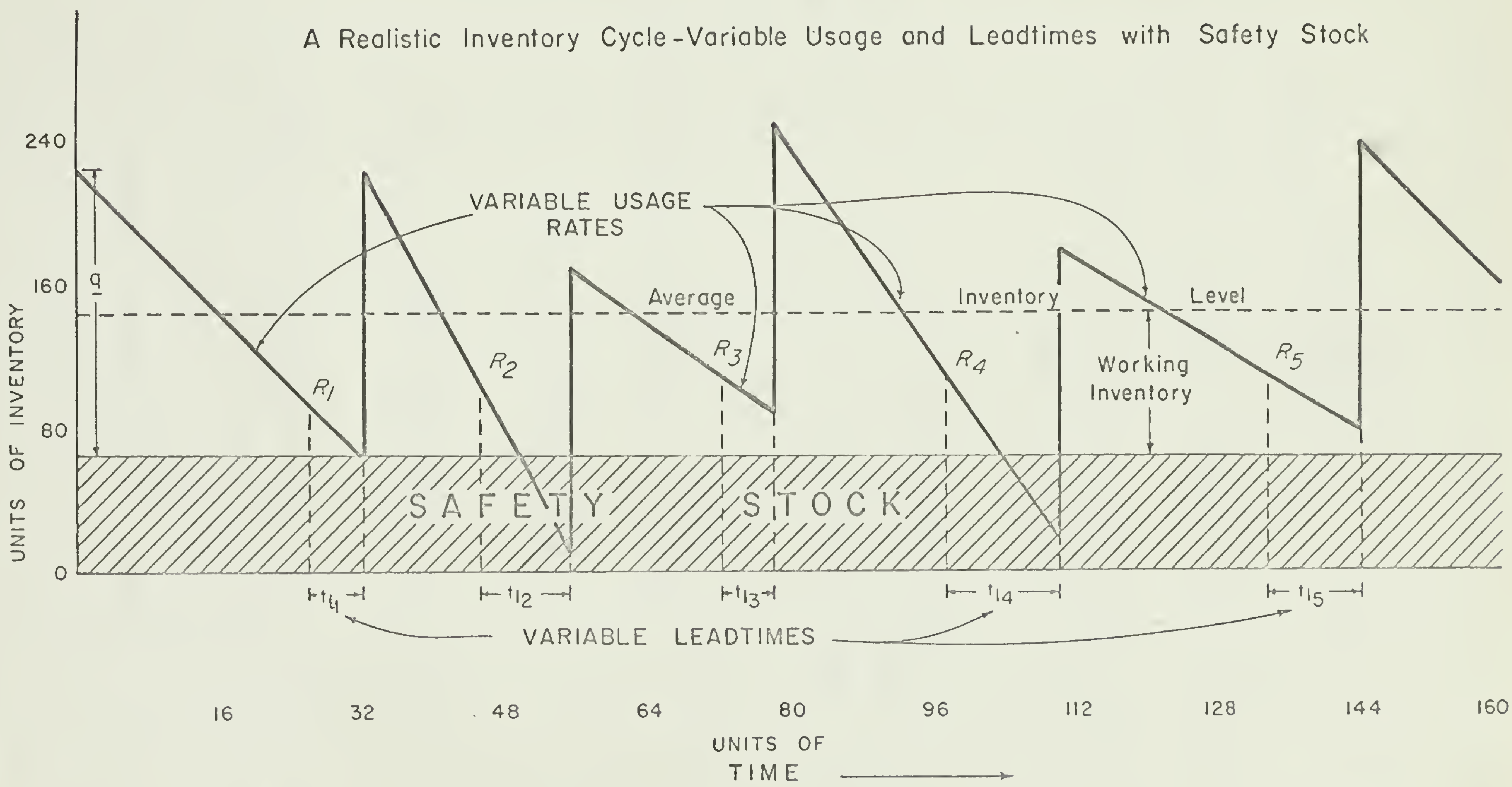
$$INV = a_o + a_1 t \ , \qquad \text{(II-2)}$$

and if we consider time to start at zero, i.e. $T_1 = 0$, then

$$INV_{avg.} = \frac{\int_0^T (a_o + a_1 t)dt}{(T - 0)} \qquad \text{(II-3)}$$

FIGURE II - 4

A Realistic Inventory Cycle-Variable Usage and Leadtimes with Safety Stock



Integrating the above expression over the limits $t = 0$ to $t = T$ we get,

$$INV_{avg.} = \frac{a_0 T + \frac{a_1}{2} T^2}{T - 0} = a_0 + \frac{a_1}{2} T \qquad \text{(II-4)}$$

This is the integrated average inventory value over the time interval of zero to T.

(Note: a_1 represents the slope of the demand curve and its value is negative in the sense that demand reduces inventory level.)

It can be seen from equation II-2 that evaluating the function at the parameters $t = 0$ and $t = T$ gives point values of the inventory at particular instants of time. At $t = 0$, $INV = a_0$. At $t = T$, $INV = a_0 + a_1 T$.

The arithmetic mean, or average of inventory, between beginning and terminal inventory is simply

$$INV_{avg.} = \frac{INV_0 + INV_T}{2}$$

$$INV_{avg.} = \frac{a_0 + (a_0 + a_1 T)}{2} = a_0 + \frac{a_1 T}{2} \qquad \text{(II-5)}$$

The above proof indicates that for a linear function, such as was assumed for the variable INV, the integrated average of the function over an interval of time (Eq. II-4) equals the arithmetic average of the terminal points (Eq. II-5). However, if the function were not linear, then the simple averaging of inventory at the terminal points, INV_0 and INV_T, would not yield a true value averaged over time.

Referring to Figure II-4, the level of inventory at any point in time can be expressed in general form as:

$$INV = a_0 + a_1 t \qquad \text{(II-2)}$$

and at time $t = 0$, $INV = a_s + q$; where a_s = safety stock and q = order quantity; at $t = T$, $INV = a_s$

Then $$a_0 = a_s + q \qquad \text{(II-6)}$$

but $$a_s = a_0 + a_1 T \qquad \text{(II-7)}$$

Therefore $$a_0 = a_0 + a_1 T + q$$

and $$a_1 = - q/T \qquad \text{(II-8)}$$

Substituting this expression for "a_1" into eq. (II-2) and considering that $a_0 = a_s + q$ we get

$$INV = a_s + q - \frac{q}{T} t \qquad \text{(II-9)}$$

To find the average level of inventory we then integrate this expression for INV over the interval $t = 0$ to $t = T$.

Therefore:

$$INV_{avg.} = \frac{\int_0^T (a_s + q - \frac{qt}{T})\, dt}{T - 0} \qquad \text{(II-10)}$$

$$= \frac{T(a_s + q)\Big|_0^T + (- \frac{q}{T} \frac{t^2}{2})\Big|_0^T}{T}$$

$$= a_s + q - \frac{q}{2}$$

$$INV_{avg.} = a_s + \frac{q}{2} \qquad \text{(II-11)}$$

where q/2 is the "average working inventory." As illustrated in Figure II-4, the average inventory level is equal to the sum of the safety stock and working inventory.

Since the term average working inventory is used so extensively in subsequent sections of this thesis, the author felt that it's rigorous derivation through the use of calculus was warranted. That the term q/2 could be arrived at by simple arithmetic averaging is valid only because the Inventory function is considered to be linear. In more complex inventory models, the same method of determination of average inventory and working inventory levels would prove equally valid, but not so the short-cut of simple arithmetic averaging.

Reorder Points - When to Reorder.

In determining a reorder level for any given SKU of merchandise, Exchange personnel need only follow the format spelled out in the Stock Control Handbook[24].

For Example:

Reorder Point level = sales X (Delivery Time + Safety Factor)

= 200/wk X (2 wks + 1 wk) = 600 units

Once this simple arithmetic operation is completed and the reorder level posted to the applicable stock card for future reference, the job is done. It could hardly be less complicated. Yet therein lies an important element of efficient inventory. Too high a reorder level, and overstocking results with its attendant financial problems. Efficient inventory

management ceases when inventory levels rise higher than necessary. On the other hand, too low a reorder level for the particular situation results in stockouts, lost sales, and patron dissatisfaction.

Just as the proper reorder level provides a boost toward efficient inventory management, a realistic determination of the safety factor provides the key to establishing this level. Although the stock control handbook leaves no doubt regarding the determination of both reorder level and safety factor, the significance of their roles in scientific inventory management warrants a more detailed analysis of these important factors.

First, it is necessary to recognize the time lag between placement of an order and the receipt of same. In allowing for this, the average expected sales during leadtime are determined. If the reorder point is set at this quantity, however, stockouts will occur in approximately 50% of the order cycles since it could be expected that actual sales would exceed average sales during half of the leadtime periods. Then too, the leadtimes will vary up or down from the average in approximately 50% of reorders, although leadtime is usually more predictable than demand. Figure II-4 illustrates both of these points. The average leadtime is 9 time units, and average usage during leadtime is 54 units of stock. In three of the five order cycles, the leadtime exceeded the average (10, 11 and 13); during two of the five, usage exceeded the average (90 and 100).

The reorder point formula is then:

$$P_R = B + \overline{S}_w \, \overline{t}_{\ell w} \tag{II-12}$$

where: P_R = reorder point

B = buffer or safety stock

$\overline{S}_w$ = Average issues (Sales) per week

$\overline{t}_{\ell w}$ = Average leadtime expressed in weeks

This formula will produce a reorder point in terms of dollars or units of stock depending upon the manner in which B and S_w are expressed.

Buffer Stock.

Although buffer stock provides a measure of safety from varying leadtimes as well as varying demands during leadtime, it is the latter factor which has the greatest effect on the level of buffer stock maintained. Manufacturers, for the most part, are prompt in shipment of their products and frequently ship in advance of specified dates. Demand during leadtimes cannot be predicted with any degree of certainty, however. The safety factors appearing in the Navy Exchange stock control handbook[25] appear to be quite arbitrary, but in reality are not.

Buffer stocks are evaluated in terms of the level of service it is desirable, or necessary, to maintain. It would be totally impractical to establish a level of buffer stock which would guarantee an in-stock position 100% of the time. Providing for unusual combinations of heavy demand,

coupled with an extra long leadtime, would result in over-stocking. However, small deviations from average in regard to demand and leadtime must be expected and provided for, if stock-outs are to be avoided.

The simplest approximation of buffer stock, which in Navy Exchanges is expressed in terms of number of weeks stock, is based on the mean value and standard deviations of the demand distribution. For a normal distribution with a mean of $\overline{S}_w$ and a standard deviation of δ, the range[26]

$$\overline{S}_w - \delta \leq S_w \leq \overline{S}_w + \delta$$

where S_w = sales in any given week (demand)

$\overline{S}_w$ = average weekly sales.

includes about 68 percent of all values of S_w; the range,

$$\overline{S}_w - 2\delta \leq S_w \leq \overline{S}_w + 2\delta$$

includes about 95 percent of all values of S_w; and the range,

$$\overline{S}_w - 3\delta \leq S_w \leq \overline{S}_w + 3\delta$$

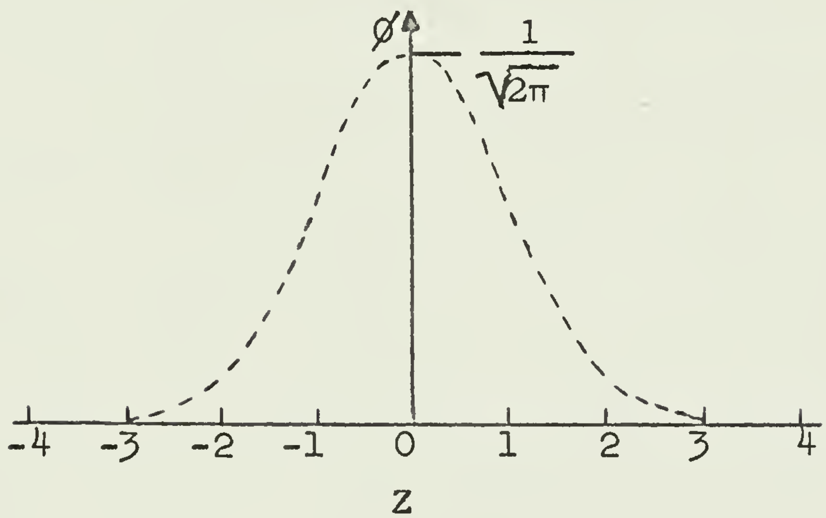
includes 99.7 percent of all values of S_w.

However, in planning buffer stock levels, only those values of S_w which exceed $\overline{S}_w$ must be considered since they represent above average demands which lead to stockouts. Consequently, we are concerned with only half of the normal demand distribution curve. (Ref. Figure II-5).

In Figure II-5, the horizontal axis is measured in units of δ_{S_w}, while the vertical axis gives the probability

FIGURE II-5

NORMAL DISTRIBUTION OF DEMAND



$$Z = \frac{(s_w - \overline{s_w})}{\delta_{s_w}}$$

that a fraction lies within the range between S_w and $(S_w + dS_w)$. Due to the symmetry of this curve, the ranges from zero include the following percentage of all values of S_w:[27]

$$0 \leq S_w \leq \overline{S}_w + \delta \text{ includes 84 percent}$$

$$0 \leq S_w \leq \overline{S}_w + 2\delta \text{ includes 97.7 percent}$$

$$0 \leq S_w \leq \overline{S}_w + 3\delta \text{ includes 99.9 percent}$$

Now if $\overline{S}_w \overline{t}_{\ell_w}$ is the mean demand during leadtimes, δ_ℓ is its standard deviation and from Eq. II-12, the reorder point P_R is:

$$P_R = \overline{S}_w \overline{t}_{\ell w} + B \qquad \text{(II-12)}$$

$$= \overline{S}_w \overline{t}_{\ell_w} + K\delta_\ell \qquad \text{(II-13)}$$

For a normal distribution[28]

if K = 1, P_R will be exceeded by demand only 16 percent of the time

if K = 2, P_R will be exceeded by demand only 2.3 percent of the time

if K = 3, P_R will be exceeded by demand only 0.1 percent of the time.

"Must" and "never-out" items require a high degree of protection from stockouts, and consequently a high value of K is assigned. The table of safety stock shown on page II-13 of the stock control handbook automatically provides a buffer stock as a function of average leadtimes for the vast majority of items ordered. However, in the case of "must" and "never-out" items the reorder point should be calculated using equation (II-13) above. A K = 3 would be appropriate for "must" items guaranteeing a 99.5% in-stock position, and a K = 1.64 would give a measure of protection to "never-out" items equal to a 95% in-stock position.

CHAPTER III

THE ECONOMIC ORDER QUANTITY CONCEPT

Historical Background.

The first public utterance on the subject of an "Economic Order Quantity" (EOQ) came in the form of a paper written by R. H. Wilson entitled, "A Scientific Routine for Stock Control", published in the Harvard Business Review in 1934.[29] Although developed independently a few years earlier by Western Electric Corporation economists, and used internally in the scientific management of their spare parts inventory, Wilson is generally credited with being the originator of the EOQ Concept.

Simply stated, the formula is a mathematical expression through the use of which we may determine that order quantity which will minimize the total annual variable costs of managing inventory. In developing his now famous EOQ formula, Wilson worked with a mathematical model which assumed that the total cost of managing inventory is made up of two parts; ordering cost and carrying cost.

Thus, as expressed by Wilson,[30]

$$Q = \sqrt{\frac{2LA}{I}} \qquad \text{(III-1)}$$

L = Average cost of handling an order in dollars.

A = Annual usage in dollars.

Where:

I = Inventory carrying cost expressed as a decimal. (i.e. % per annum of working inventory level)

Q = Economic Order Quantity.

Note: the above symbols are those used in the original paper.

Originally devised as a scientific tool to assist in the management of industrial inventories, the EOQ concept was immediately recognized as being equally applicable to the retail trade when ordering staple year-round merchandise. However, the depression year of 1934 found retailers in the throes of "Hand-to-mouth" buying[31]. Afraid to be caught with large inventories in a time of sharply reduced buying, merchants ordered frequently while maintaining inventory at the lowest possible levels. In maximizing order frequency "across the board", retailers could not take advantage of the cost savings resulting from ordering in economic order quantities. As pointed out in Chapter I, scientific inventory control methods, and in particular the EOQ concept, did not take hold in the retail trade until some fifteen years ago.

Traditional Mathematical Approach.

In developing the EOQ formula, the two variable inventory management cost groups comprising total variable cost can be expressed as follows:

Total Variable Cost = order cost + carrying cost

or

$$TVC = NC_o + \frac{SI}{2N} \qquad \text{(III-2)}$$

N = Annual number of orders cut

where: C_o = Cost per order per item ($)

S = Annual Sales ($)

I = Inventory carrying cost expressed as a decimal.

but

$$N = \frac{S}{q},$$

where: q = order quantity in dollars

Therefore, equation III-2 can also be expressed in the form:

$$TVC = \frac{SC_o}{q} + \frac{qI}{2} \qquad \text{(III-2A)}$$

In order to minimize total variable costs (TVC) in the above equation, we differentiate with respect to the independent variable "q", set the derivative equal to zero, and solve the resulting equation.[32] This gives:

$$\frac{d(TVC)}{d(q_{opt})} = 0 = -\frac{SC_o}{q_{opt}^2} + \frac{2I}{4} \qquad \text{(III-3)}$$

$$q_{opt}^2 = Q^2 = \frac{2SC_o}{I}$$

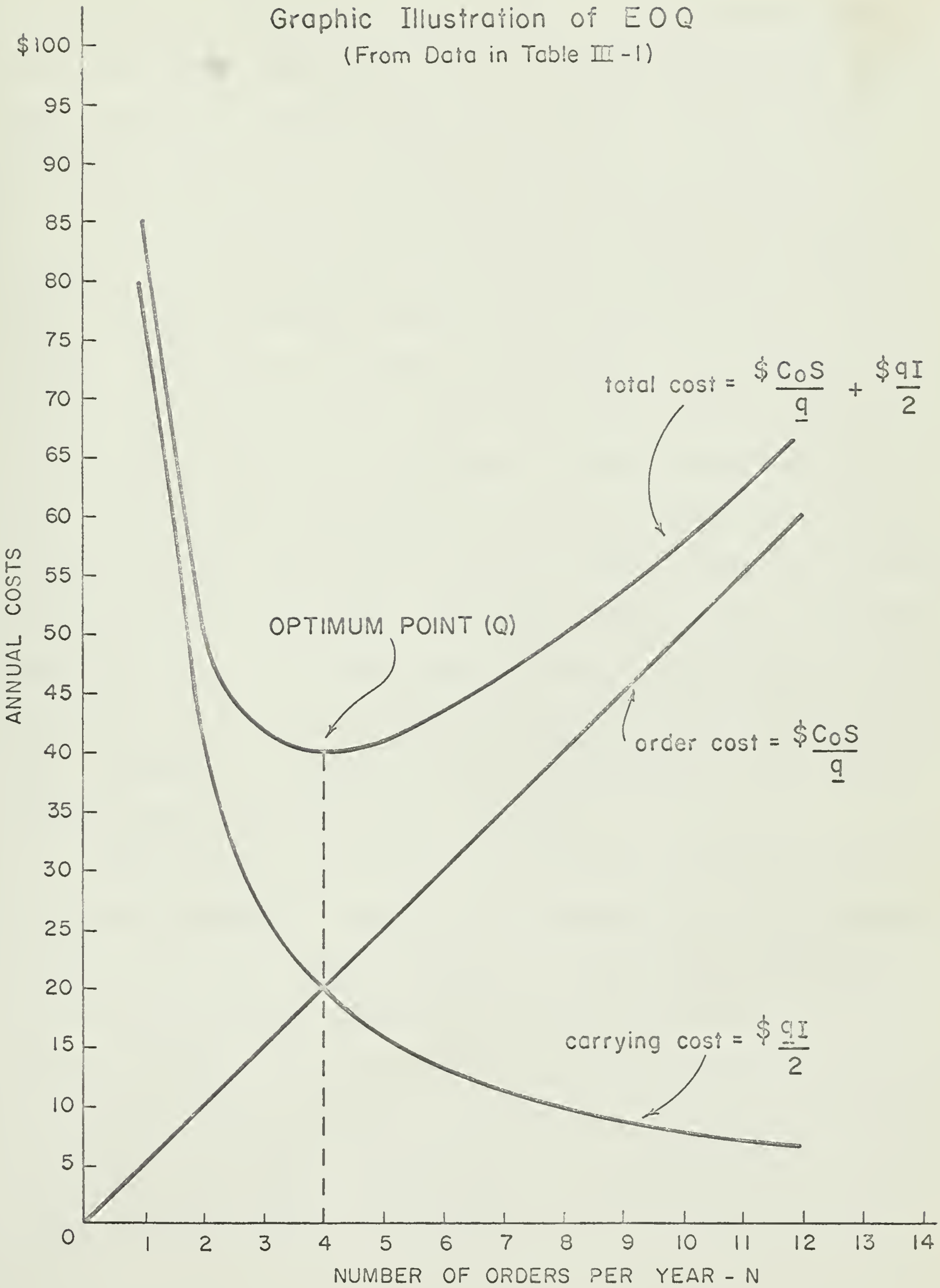
and solving for Q:

$$Q = \sqrt{\frac{2C_oS}{I}} \qquad \text{(III-3A)}$$

Equation III-3A is the Economic Order Quantity formula developed by Wilson, and is identical to equation III-1 except for the different symbolism employed.

Figure III-1 provides a graphic illustration of the Economic Order Quantity for a particular unit of merchandise

FIGURE III-1


Graphic Illustration of EOQ
(From Data in Table III-1)
$100
95
90
85
80
75
70
65
60
55
50
45
40
35
30
25
20
15
10
5
0
ANNUAL COSTS
total cost = $\$\frac{C_oS}{q} + \$\frac{qI}{2}$
OPTIMUM POINT (Q)
order cost = $\$\frac{C_oS}{q}$
carrying cost = $\$\frac{qI}{2}$
1
2
3
4
5
6
7
8
9
10
11
12
13
14
NUMBER OF ORDERS PER YEAR - N

in a Navy Exchange. The inventory model depicted in this illustration is shown in Table III-1. Actual carrying costs and order costs applicable to the Navy Exchange Program have been assigned[33]. These costs are 10% per annum of working inventory level and $1.28 per item per order respectively.

The total cost line is simply a summation of the individual cost curve values at any given level of N, the number of orders processed annually for the given item. It is readily apparent that in this illustration, as in most real inventory cost situations, the total cost line has only one minimal point where the derivative of its function is zero. This point represents the minimum total cost. An important point to be noted is that for some distance on either side of the minimum point, the curve flattens out. An order cycle of 3, 4, or 5 orders per year would produce very little difference in total inventory management costs of this particular commodity, although an order frequency of 4 per annum produces the "Economic Order Quantity."

The purpose of the EOQ formula is to minimize total inventory management costs by determining that order quantity which attains the optimum balance between annual ordering and carrying costs. However, it is not always practical to order the exact quantities dictated by a rigid application of the formula. The fact that in most real inventory situations, the total cost curve flattens out at its minimum point provides a useful degree of flexibility in choosing the most

TABLE III-1

Relationship of Annual Ordering Cost and Carrying Cost to Total Inventory Management Cost in a Single Item Inventory.

ASSUMPTIONS: S = $1600.00 C_o = $5.00 per item per order I = .1

No. Orders Per Year(N)	Annual Order Cost $(\frac{C_o S}{q})$	Order Quantity(q)	Working INV. (q/2)	Annual Carrying Cost $(\frac{qI}{2})$	Total Annual Inventory Costs $(\frac{C_o S}{q} + \frac{qI}{2})$
1	5.00	1600.00	800.00	80.00	85.00
2	10.00	800.00	400.00	40.00	50.00
3	15.00	533.33	266.67	26.67	41.67
4	20.00	400.00(EOQ)	200.00	20.00	40.00
5	25.00	320.00	160.00	16.00	41.00
6	30.00	266.67	133.33	13.33	43.33
7	35.00	228.57	114.28	11.43	46.43
8	40.00	200.00	100.00	10.00	50.00
9	45.00	177.78	88.89	8.89	53.89
10	50.00	160.00	80.00	8.00	58.00
11	55.00	145.45	72.72	7.27	62.27
12	60.00	133.33	66.67	6.67	66.67

practical, as opposed to the most economic, order quantity without incurring too great a cost penalty, provided, of course, that the difference in the two order quantities is not excessive.

In compiling the "order factor tables" published in the supplement to the Stock Control Handbook , the Navy Ships Store Office[35] specialists have taken advantage of this "curve flattening" phenomena. Instead of calculating a specific economic order quantity for each sales level, a range of sales volumes is established and order quantities are expressed in terms of months or weeks usage (order factors) for items within that range. For example, in the sales range of $300.01 to $600.00 per annum, an order quantity of 3 months usage would be placed each time an item within that range is reordered. For an item with annual sales of $300.01, the order quantity would be calculated as:

$$q_{\$300.01} = 3 \frac{\text{mos. sales}}{\text{order quantity}} \times \frac{\$300.01}{12 \text{ mos. sales}}$$

$$= \$75.00/\text{order quantity}$$

and for the item with annual sales of $600.00:

$$q_{\$600.00} = 3 \frac{\text{mos. sales}}{\text{order quantity}} \times \frac{\$600.00}{12 \text{ mos. sales}}$$

$$= \$150.00/\text{order quantity}$$

Applying the EOQ formula (Eq. III-3A) to these calculations:

$$Q_{\$300.01} = \sqrt{\frac{2 \times 1.28 \times 300.01}{.1}} = \$87.64/\text{order quantity}$$

and

$$Q_{\$600.00} = \sqrt{\frac{2 \times 1.28 \times 600.00}{.1}} = \$123.94/\text{order quantity}$$

At the low end of each sales range, the order factor tables order quantity will be lower than that calculated by the EOQ formula and at the upper limit it will be higher. This variation in order quantities calculated by the two methods explains the difference in inventory levels and total variable inventory management costs depicted in Table IV-1.

Problems Encountered in Adopting EOQ.

When a retail firm, such as a Navy Exchange, changes its order policy from one of a monthly or quarterly order cycle to an EOQ order basis, differences in working inventory levels and number of orders processed annually will appear. If on a monthly order cycle, it is likely that working inventory level will increase, while numbers of orders processed annually will drop sharply[36]. If operating previously on a quarterly order cycle, the reverse situation may appear at time of changeover. In either event, adjustments will have to be made to handle the increased workload in the applicable functional area. If the operation is sufficiently flexible in terms of readily available unused warehouse space, and trained personnel to staff either a larger warehouse or an expanded procurement section, the changeover can be accomplished with little difficulty. However, if the firm does not enjoy such a high degree of flexibility in its warehouse and procurement functions, serious problems can result.

Obviously, if warehouse space is barely adequate for a particular level of inventory and there is no possibility of obtaining additional space, it would be impractical to adopt an order policy that would result in an increased inventory level. Although additional staffing of a procurement section to handle an increased workload is undoubtedly easier to achieve than obtaining additional warehouse space where none exists, it may have to be accomplished on an incremental basis over a fairly lengthy time span.

In either case, it would be highly beneficial if a compromise could be found in the form of a more flexible order policy than that offered by EOQ. Such an order policy would seek an optimum relationship between working inventory level and numbers of orders processed while operating under imposed restraints on either inventory level or number of orders processed or both, dictated by the practical limitations of the particular operation. The ultimate goal would be complete cost optimization, but it would be arrived at on an incremental basis. Subsequent sections of Chapter III will develop the basis for such a flexible order policy.

Optimum Relationship - Inventory Vs. Orders.

Restating the EOQ formula of Eq. III-3A in a slightly different form:

$$Q = \sqrt{\frac{2C_o}{I}} \times \sqrt{S} \qquad \text{(III-4)}$$

and, since average working inventory is one-half the order quantity when sales are a linear function:

$$Q = 2 \times INV_{opt.} = \sqrt{\frac{2C_o}{I}} \times \sqrt{S} \qquad \text{(III-5)}$$

In any given retail organization, the values of C_o and I can be considered as constants[37]. Eq. III-5 then becomes:

$$2 \times INV_{opt.} = Q = K \times \sqrt{S} \qquad \text{(III-5A)}$$

where

$$K = \sqrt{\frac{2C_o}{I}} \qquad \text{(III-5B)}$$

Equation (III-5A) can in turn be expressed in the form of a rule, referred to in future sections as Rule I.

"In a given inventory, the optimum relationship exists between average working inventory and the total number of purchase orders cut when the individual order quantities are a constant times the square root of the individual sales."[38]

Before proceeding further, it would be well to devise a model inventory - order situation. In this context, the model is a representation of the behavior of an actual segment of inventory. If the model is an adequate representation of the order-inventory operation, it is possible to learn how to improve the operation by experimenting with the model.

For the sake of simplicity, a three SKU inventory will be assumed to be composed of an item of merchandise from each of the three classes (by sales volume) illustrated

in Chapter II. Relationships proved valid in this simplified inventory model would prove equally valid in a 300 or 10,000 SKU inventory, provided that there is no appreciable difference in order costs or carrying costs of the units involved. That this point is true in most inventory situations is borne out by the statement of Buchan and Koenigsberg that, "in general, both the ordering and carrying cost percentage are a function of the company rather than of the inventory item; in other words, these factors tend to be constant for all items of a particular inventory."[39]

Table III-2 illustrates this greatly simplified three SKU inventory, in which orders are placed on a non-optimum monthly order cycle, which was the normal order policy in Navy Exchanges prior to 1964. The total annual inventory cost for this model is $188.75. A cost conscious inventory manager would immediately ask the questions, 1. "Does this inventory model represent the optimum situation?," and 2. "if not, what steps can be taken to reduce total management costs?"

The Conflict.

A quick inspection of just item A is sufficient to answer the first of these questions in the negative.

$$Q = \sqrt{\frac{2C_oS}{I}} \qquad \text{(III-3A)}$$

$$Q = \sqrt{\frac{2C_o}{I}} \times \sqrt{S}$$

$$Q = \sqrt{\frac{2 \times 5.00}{.1}} \times \sqrt{1600.} = 10 \times 40 = \$400.00$$

TABLE III-2

A Three Item Inventory on a Monthly Order Cycle

Item	S	N	q	INV. (q/2)
A	$1600.00	12	$133.33	66.67
B	400.00	12	33.33	16.67
C	100.00	12	8.33	4.16

$\sum N = 36 \quad \sum q = \$174.99 \quad \sum INV = 87.50$

Assumptions: $C_o = \$5.00$ per item per order $I = .1$

Annual order costs = $\$NC_o = 36 \times 5.00 = \$180.00$

Annual carrying costs = $\sum q/2 \times I = 87.50 \times .1 = 8.75$

Total Annual Inventory Costs = $188.75

TABLE III-3

Optimization of Total Inventory Management Costs of Three Item Inventory Depicted in Table III-2

Item	S	$\sqrt{S}$	$\sqrt{\frac{2C_o}{I}}$	Q	N	INV(Q/2)
A	$1600.00	40.00	10.0	$400.00	4	$200.00
B	400.00	20.00	10.0	200.00	2	100.00
C	100.00	10.00	10.0	100.00	1	50.00
	$\sum$	70.00		$700.00	7	$350.00

Assumptions: $C_o = \$5.00$ per item per order $I = .1$

Annual order costs = $\$NC_o = 7 \times 5.00 = 35.00$

Annual carrying costs = $\sum Q/2 \times I = 350.00 \times .1 = 35.00$

Total Annual Inventory Costs = $70.00

Comparing the above calculated economic order quantity with that of Table III-2 it is found that:

$$Q(\$400.00) \neq q(\$133.00)$$

Therefore, it can be concluded that item A is not being ordered in the most economical quantity. Since optimization of total inventory costs depends upon suboptimization of its individual item component costs, the inventory model in question does not represent an optimum cost situation. Though the ultimate goal is always to minimize total costs, it may be impractical, or impossible to do so immediately, and a comparison of working inventories shown in Table III-3 with that of Table III-2 provides the clue.

In optimizing (i.e. minimizing) annual inventory management costs, the total number of orders placed annually have been reduced from 36 to 7. This is a welcome reduction in work load on the Purchase Office Staff, some of whom may be utilized efficiently elsewhere so as to realize the economies in order costs which are indicated by the 80.7% reduction in order frequency. However, working inventory has shot upward drastically to a level of \$350.00 from \$87.50; a four-fold increase dictated by the economic order quantity formula.

In terms of total costs, the net result is a highly favorable decrease of 62.9%, yet the practical aspects of such a change might prove to be completely impossible.

Almost four times as much warehouse space would be required to store the larger inventory, and its immediate availability would be very rare, indeed. Although the example shows a much higher inventory increase than would be expected in a real situation, it was deliberately chosen to illustrate an obstacle that might well be encountered when shifting to EOQ ordering from a regular monthly reorder cycle of order placement. This situation may become a very real problem in those Exchanges having a minimum of storage space available. Hence, the statement in the SIM Handbook that shifting to the EOQ Concept will reduce expenditures of inventory management dollars, --- "At Exchanges with adequate storage facilities."[40]

Immediate Cost Reduction Alternatives.

Considering the inventory situation depicted in Table III-2, there are three practical courses of action which can be taken to reduce total inventory management costs without increasing inventory levels with the attendant need for additional storage space.

1. Decrease the total number of orders cut while holding working inventory level constant at $87.50.
2. Decrease working inventory level while holding the total number of orders cut annually to 36.
3. A combination of both of these courses of action.

Table III-4 provides an example of the same three item inventory in which the working inventory level has been minimized while holding the total number of orders constant at 36. This optimum relationship between working inventory and number of orders cut is achieved by application of the rule stated on Page 37 in which:

$$K_{R.I.} = \frac{\sum \sqrt{S}}{\sum N} \qquad \text{(III-6)}$$

i.e. $$K_{R.I.} = \frac{70}{36} = 1.945 \qquad \text{(III-6A)}$$

In like manner, Table III-5 illustrates an example of the inventory for which the total number of orders cut $\sum N$ has been minimized while holding the working inventory level constant at $87.50. In this example,

$$K_{R.O.} = \frac{\sum Q}{\sum \sqrt{S}} \qquad \text{(III-7)}$$

i.e. $$K_{R.O.} = \frac{175}{70} = 2.50 \qquad \text{(III-7A)}$$

Note: the mathematical proof of Equations III-6 and III-7 appears in Appendix B.

In the first example, inventory has been reduced 22.2% without any increase in the total number of orders cut, and total costs have been reduced by $1.94. In the

TABLE III-4

Reduction of Inventory Level for Given $\sum N$

$$K_{RI} = \frac{\sum\sqrt{S}}{\sum N} = \frac{\sqrt{1600} + \sqrt{400} + \sqrt{100}}{36} = \frac{40 + 20 + 10}{36} = \frac{70}{36} = 1.945$$

Item	Annual Sales S	$\sqrt{S}$	K_{RI}	q	$N = \frac{S}{q}$	INV=q/2
A	1600.00	40.00	1.945	77.80	20.55	38.90
B	400.00	20.00	1.945	38.90	10.30	19.45
C	100.00	10.00	1.945	19.45	5.15	9.72
					$\sum N$=36.00	$\sum$INV=68.07

Annual order cost = 36 x 5.00 = \$180.00

Annual carrying cost = \$68.07 x .1 = 6.81

Total inventory management cost = \$186.81

Savings over original model = \$188.75 - 186.81 = \$1.94

TABLE III-5

Reduction of Number of Orders for Given Inventory Level

$$K_{RO} = \frac{\sum Q}{\sum\sqrt{S}} = \frac{133.33 + 33.33 + 8.33}{\sqrt{1600} + \sqrt{400} + \sqrt{100}} = \frac{174.99}{70.00} = 2.5$$

Item	S	$\sqrt{S}$	K_{RO}	q	N	INV
A	1600.00	40.00	2.5	100.00	16	50.00
B	400.00	20.00	2.5	50.00	8	25.00
C	100.00	10.00	2.5	25.00	4	12.50
					$\sum N$ = 28	$\sum$INV = 87.50

Annual order cost = 28 x 5.00 = \$140.00

Annual carrying cost = 87.50 x .1 = 8.75

Total inventory management cost = \$148.75

Savings over original model = \$188.75 - 148.75 = \$40.00

second example, the total number of orders has also been decreased by 22.2% without an increase in inventory level, and total cost has been reduced \$40.00. By using values of K between 1.945 and 2.5, both inventory level and total number of orders will be reduced. For example, with K = 2.2, working inventory amounts to \$77.00 with a requirement that a total of only 32 orders be prepared annually, thereby permitting a total cost savings of \$21.05.

Considering the original inventory model, with assumptions made pertaining to C_o and I of \$5.00 per item ordered and 10% per annum of inventory dollars respectively, it is obvious that a reduction in the number of orders cut has a much greater impact on total costs than does a corresponding decrease in working inventory levels. However, it has been shown that reducing order frequency to its most economic level could impose very serious warehousing problems. Therefore it may be necessary to approach the economic order quantity levels of $\sum INV$ and $\sum N$ incrementally rather than in a single jump. How this logical approach towards optimization can be accomplished will be demonstrated in a subsequent section.

<u>The Optimal Policy Curve.</u>

Since both conditions expressed by equations III-6 and III-7 are true under optimum conditions of order quantities under Rule 1, we can equate the right side of these two equations. It can therefore be said that at $q_{optimum}$, i.e. Q,

$$K_{R.O.} = K_{R.I.} \qquad \text{(III-8)}$$

Substituting the mathematical expressions from equations III-6 and III-7 for $K_{R.O.}$ and $K_{R.I.}$ respectively into Eq. III-8:

$$\frac{\sum Q}{\sum \sqrt{S}} = \frac{\sum \sqrt{S}}{\sum N} \qquad \text{(III-9)}$$

and

$$\sum N \times \sum Q = (\sum \sqrt{S})^2 \, . \qquad \text{(III-10)}$$

Since

$$\sum Q = 2 \sum INV \qquad \text{(III-11)}$$

substitution of $2\sum INV$ for $\sum Q$ into equation III-10 produces:

$$\sum N \times 2 \sum INV = (\sum \sqrt{S})^2 \qquad \text{(III-12)}$$

$$\sum N \times \sum INV = \frac{(\sum \sqrt{S})^2}{2} \qquad \text{(III-13)}$$

or, for the example given:

$$\sum N \times \sum INV = \frac{(70)^2}{2} = 2450 \qquad \text{(III-14)}$$

Equation III-14 establishes a boundary concept for the order numbers-working inventory relationship for the example chosen. From this equation, Table III-6 has been compiled. The data from Table III-6, in turn, has been plotted in Figure III-2 to produce a boundary curve, i.e. the Optimal Policy Curve.[41] Each point on this curve

TABLE III-6

Working Inventory Versus Order Numbers Relationship

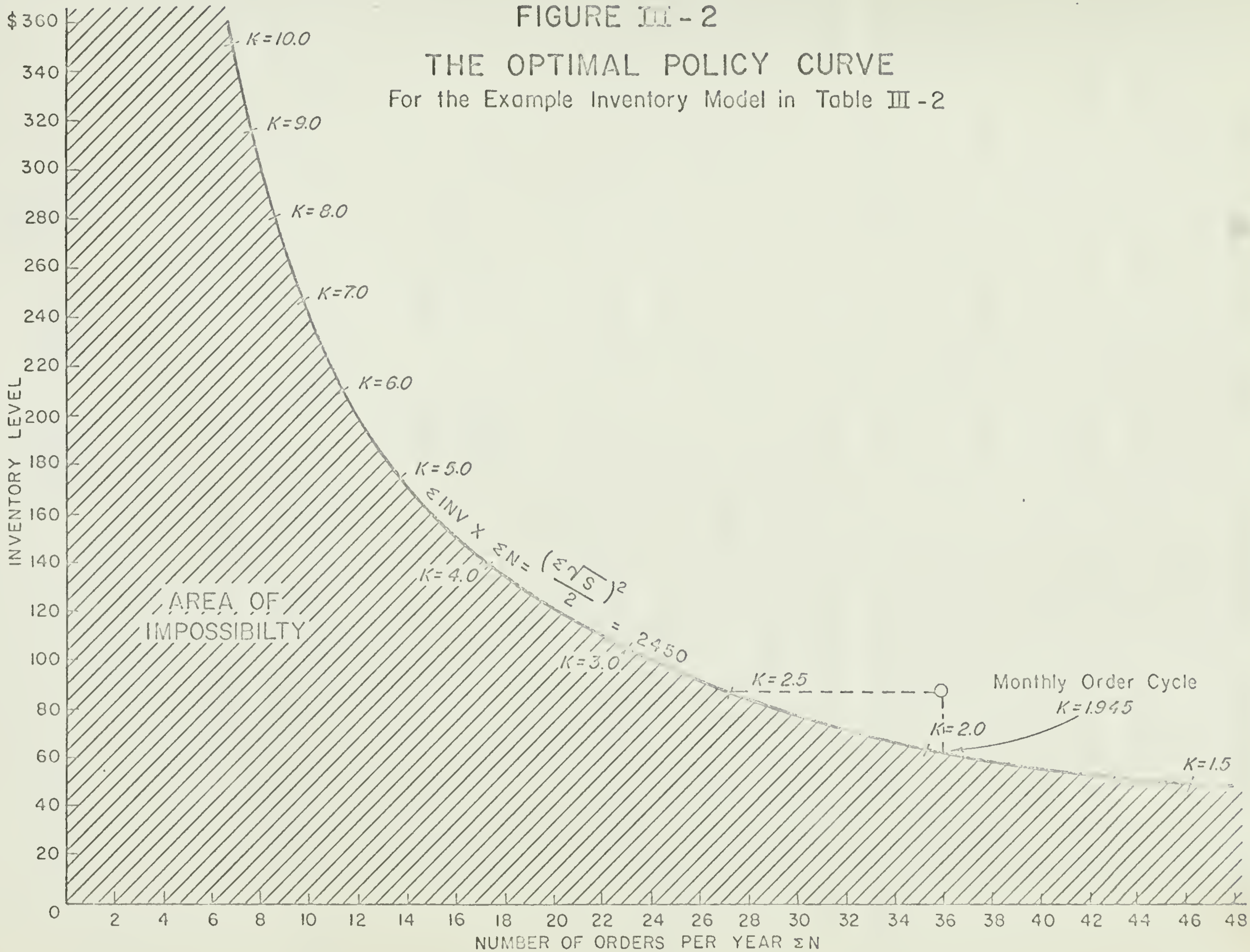
$$\sum N \times \sum INV = 2450 \quad \text{Eq. III-14}$$

$\sum N$	$\sum INV$	$\sum N$	$\sum INV$
2	$1225.00	26	94.30
4	612.50	28	87.50
6	408.33	30	81.66
8	306.25	32	76.56
10	245.00	34	72.20
12	204.16	36	68.06
14	175.00	38	64.60
16	153.12	40	61.75
18	136.11	42	58.30
20	122.50	44	55.70
22	111.40	46	53.30
24	102.08	48	51.04

FIGURE III - 2

THE OPTIMAL POLICY CURVE

For the Example Inventory Model in Table III - 2



represents an optimum relationship between working inventory and total numbers of orders cut.

The area to the left of the curve represents impossible combinations of working inventory and total numbers of orders for this particular inventory model. For example, it is impossible to cut only 20 purchase orders per year and have a working inventory level of $100.00. For this number of purchase orders, the minimum possible inventory level would be $122.50, the point at which the vertical line N = 20 intersects the Optimal Policy Curve. On the other hand, all combinations of orders and inventory represented by points to the right of the curve are possible, though non-optimal combinations. For example, a combination of N = 20, INV = $300.00 is a possibility. (Refer to Table III-7 for calculations showing non-optimal nature of this relationship.)

Equation III-7 can be written as,

$$K = \frac{\sum Q}{\sum \sqrt{S}} = \frac{2\sum INV}{\sum \sqrt{S}} \qquad \text{(III-15)}$$

and since the term $\sum \sqrt{S}$ is a known constant at $70.00 for the given inventory model, the boundary curve delineating the optimum order-inventory relationship can readily be calibrated for all values of K, where

$$K = \frac{2\sum INV}{\sum \sqrt{S}} = \frac{2\sum INV}{70} = \frac{\sum INV}{35} \qquad \text{(III-16)}$$

This is illustrated in Figure III-2. It was shown previously

TABLE 7

CALCULATION OF OPTIMAL ORDERS - INVENTORY RELATIONSHIPS

GIVEN: Non-optimal inventory model for which $\sum N = 20$, $\sum INV = \$300.00$

ASSUMPTIONS: $C_o = \$5.00$; $I = .1$

TVC (Non-optimal model) $= 20 \times \$5.00 + 0.1 \times \$300.00$

$= \$100.00 + \$30.00 = \underline{\$130.00}$

Calculation of TVC for optimal $\sum N - \sum INV$ relationships

$\sum N \times \sum INV = 2450$ (Ref. Table III-6)

GIVEN: $\sum N = 20$ (restraint imposed on orders)

$$\text{Then } \sum INV = \frac{2450}{\sum N} = \frac{2450}{20} = \$122.50$$

For $\sum N = 20$; $\sum INV = \$122.50$

TVC (optimal @ $\sum N = 20$) $= 20 \times \$5.00 + 0.1 \times \$122.50$

$= \$100.00 + \$12.25 = \underline{\$112.25}$

GIVEN: $\sum INV = \$300.00$ (restraint imposed on inventory)

$$\text{Then } \sum N = \frac{2450}{\sum INV} = \frac{2450}{300.00} = 8.17$$

For $\sum INV = \$300.00$; $\sum N = 8.17$

TVC (optimal @ $\sum INV = \$300.00$) $= 8.17 \times \$5.00 + 0.1 \times \$300.00$

$= 40.85 + 30.00 = \underline{\$\ 70.85}$

NOTE: Both optimal relationships result in reduced costs over their non-optimal origin.

(Eq. III-6A) that a K value of 1.945 would serve to minimize working inventory level while holding order numbers to 36. Similarly, a K value of 2.5 (Eq. III-7A) served to minimize order numbers while holding the working inventory level constant. In Fig. III-2, it can be seen that as K values decrease from 1.945, inventory levels will also decrease but at the expense of increasing order numbers. On the other hand, values of K greater than 2.5 will decrease order numbers but at the expense of increasing inventory level.

By utilizing the calibrated curve of Fig. III-2, it is possible to pick out any order number-inventory level combination desired. Simply by picking the applicable value of K from the curve, the order quantities and order frequency of each item of inventory can be quickly calculated. It is important, however, to understand that the optimal policy curve in Fig. III-2 is applicable only to the particular inventory model illustrated in Table III-2. The principle is valid for an inventory of any size, provided that the ordering costs and inventory carrying costs (%) of the various items making up that inventory can be considered constant for all items making up the inventory. However, a new curve must be constructed for each new inventory model.

Although the Optimal Policy Curve of Fig. III-2 provides an infinite number of points representing optimum relationships between inventory level and total number of orders, there is only one combination which produces optimum (i.e. minimum) total inventory management costs. If lines of

constant total costs are drawn on the Optimal Policy Curve as in Figure III-3, it is readily apparent that the "economic order quantity" solution is to be found at a K value of approximately 10.0, i.e. where a line of constant cost is tangent to the curve.

Figure III-4 provides an enlargement of this critical area. From the detailed graph, it is obvious that only one line of constant cost will be tangent to the curve, and in this model that point of tangency is exactly at a K value of 10.0. It is not merely coincidental that the $70.00 total cost represented by all points on this tangent line is the same cost computed in Table III-3 for the model in which orders were placed on an Economic Order Quantity basis. Each point on the Optimal Policy Curve represents an optimum relationship between inventory level and order numbers, but only one of these points represents that relationship which also produces optimum total costs. The point of tangency, therefore, satisfies the conditions of the Economic Order Quantity.

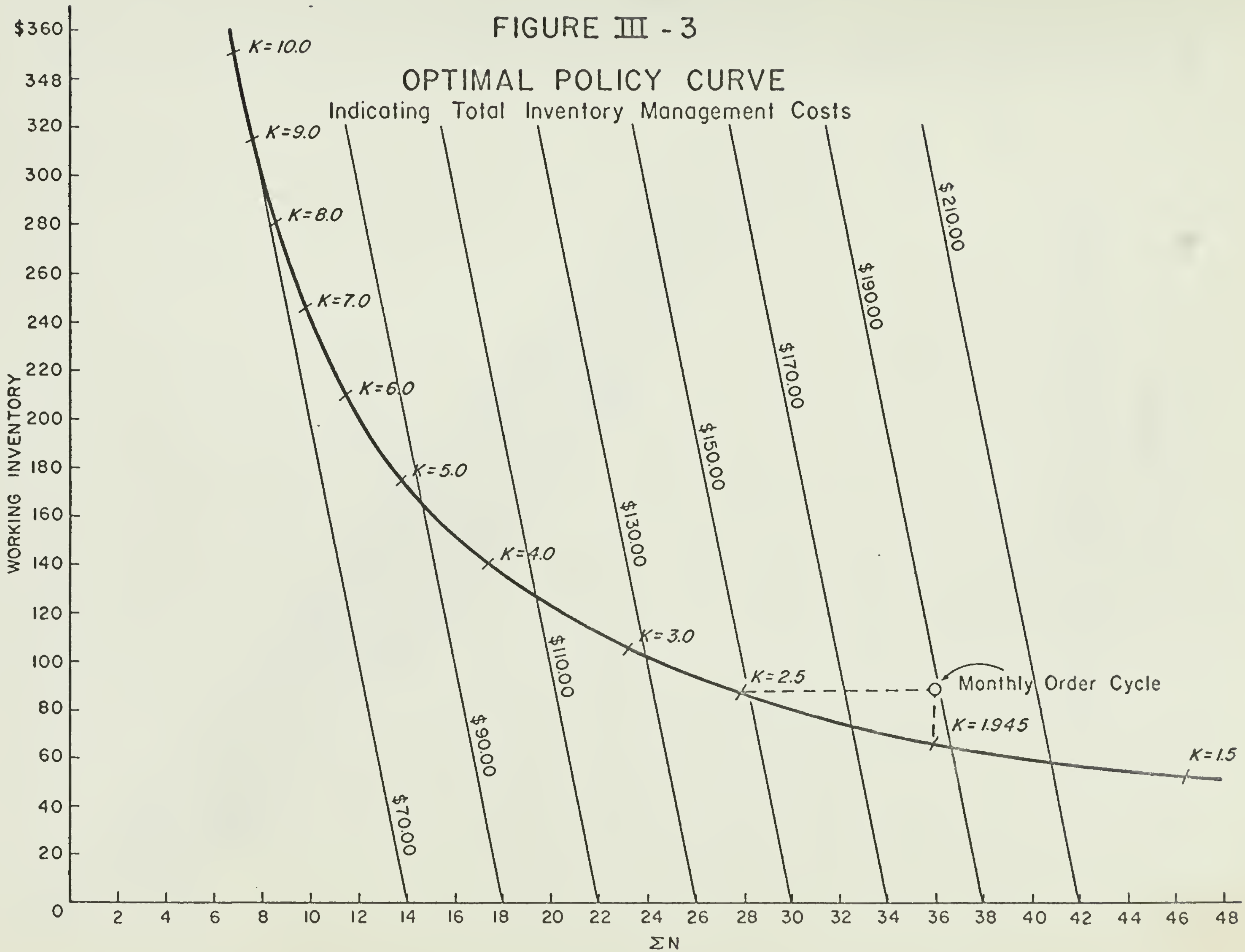
An Incremental Approach.

The path leading from the original monthly order cycle inventory situation with annual inventory management costs of $188.75, to the most efficient - from the standpoint of costs - order-inventory relationship resulting in greatly reduced costs of $70.00, may be impossible to negotiate in one step. Intermediate inventory dollar goals may have to be

FIGURE III - 3

OPTIMAL POLICY CURVE

Indicating Total Inventory Management Costs



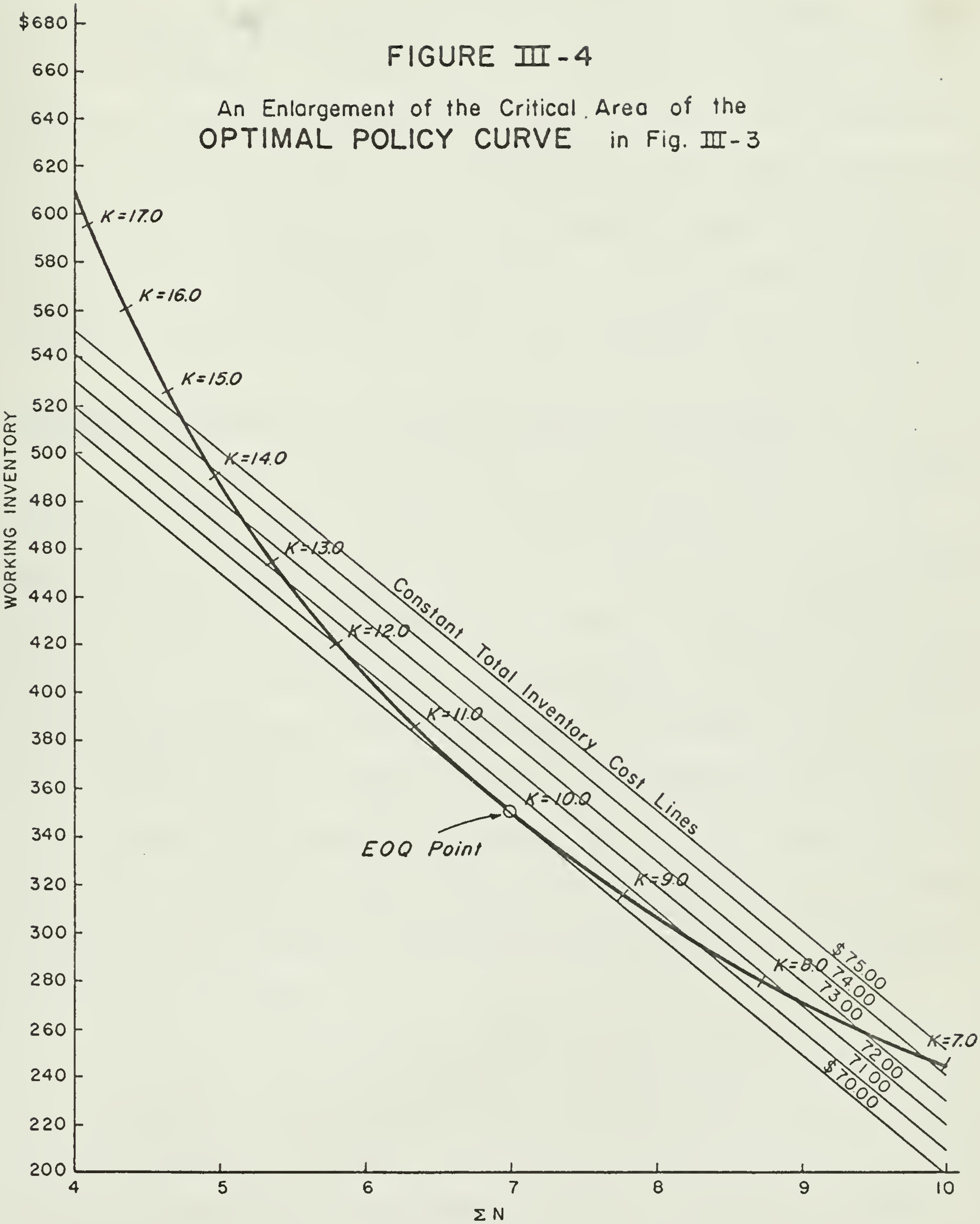


FIGURE III-4
An Enlargement of the Critical Area of the
OPTIMAL POLICY CURVE in Fig. III-3
WORKING INVENTORY
ΣN
Constant Total Inventory Cost Lines
EOQ Point
K=17.0
K=16.0
K=15.0
K=14.0
K=13.0
K=12.0
K=11.0
K=10.0
K=9.0
K=8.0
K=7.0
$75.00
74.00
73.00
72.00
71.00
$70.00
$680
660
640
620
600
580
560
540
520
500
480
460
440
420
400
380
360
340
320
300
280
260
240
220
200
4
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10

set as additional warehouse space is made available. The simple mathematical tools needed to accomplish this incremental approach toward optimization have been thoroughly explored in this chapter. For example, in the given inventory model, assuming that additional space would be available immediately to accommodate a $150.00 inventory, and additional increments of $100.00 inventory storage area would be made available in each of two subsequent years, the three step order frequency-inventory models would be as illustrated in Table III-8.

Simulated Application of the Incremental Approach.

Most retail inventories are heavily weighted in terms of numbers of SKU's in the low dollar value ranges.[42] A simulated inventory model based upon the dollar-value distribution explained in Chapter II is shown in Table III-9. It contains an item selected at random from each of nine dollar sales ranges found in Navy Exchanges.[43] These sales were then multiplied by a frequency factor[44] designed to produce an inventory-sales model of staple items which would approximate that of a very large Group VI Exchange such as NTC Great Lakes or N. S. Norfolk. The resulting model is a simulated 4490 SKU inventory.

In order to save time, which would have been consumed in lengthy hand calculations, and to demonstrate the advantage to be gained through utilization of the most advanced of Scientific Inventory Management tools, a computer program was devised to simulate the desired inventory model. This

TABLE III-8

Order Frequency - Inventory Models for Incremental Approach Toward Optimization of Costs

Time: Present Allowable Inventory Level = $150.00

$$K = \frac{\sum INV}{35} = \frac{150}{35} = 4.286$$

Item	S	$\sqrt{S}$	K	q	N	INV
A	1600.00	40.00	4.286	171.44	9.33	85.72
B	400.00	20.00	4.286	85.72	4.67	42.86
C	100.00	10.00	4.286	42.86	2.33	21.43
					$\sum N$ = 16.33	$\sum INV$=$150.01

Total INV MGT Costs = 5.00 x 16.33 + 150.00 x .1 = $96.65

Time: Plus 1 year. Allowable Inventory Level = $250.00

$$K = \frac{\sum INV}{35} = \frac{250}{35} = 7.143$$

Item	S	$\sqrt{S}$	K	q	N	INV
A	1600.00	40.00	7.143	285.72	5.60	142.86
B	400.00	20.00	7.143	142.86	2.80	71.43
C	100.00	10.00	7.143	71.42	1.40	35.71
					$\sum N$ = 9.80	$250.00

Total INV MGT Costs = 5.00 x 9.80 + 250.00 x .1 = $74.00

Time: Plus 2 years. Allowable Inventory Level = $350.00

$$K = \frac{\sum INV}{35} = \frac{350}{35} = 10.0$$

Item	S	$\sqrt{S}$	K	q=Q	N	INV
A	1600.00	40.00	10.0	400.00	4	200.00
B	400.00	20.00	10.0	200.00	2	100.00
C	100.00	10.00	10.0	100.00	1	50.00
					$\sum N$ = 7	$\sum INV$ = $350.00

Total INV MGT Costs: 5.00 x 7 + 350.00 x .1 = $70.00

TABLE III-9

A Simulated Inventory Model for a Navy Exchange
Monthly Order Cycle
Total Number of SKUS for Simulated Inventory Model = 4490.

Item Sales	Item Freq.	N	Total Orders	Order Quantity	Working Inventory
$ 100.00	1500	12	18000	$ 8.33	$ 6,250.00
$ 144.00	1000	12	12000	$ 12.00	$ 6,000.00
$ 289.00	700	12	8400	$ 24.08	$ 8,429.17
$ 441.00	500	12	6000	$ 36.75	$ 9,187.50
$ 900.00	380	12	4560	$ 75.00	$ 14,250.00
$ 1,600.00	250	12	3000	$ 133.33	$ 16,666.67
$ 4,900.00	100	12	1200	$ 408.33	$ 20,416.67
$10,000.00	50	12	600	$ 833.33	$ 20,833.33
$90,000.00	10	12	120	$7,500.00	$ 37,500.00
Totals	4490		53880		$139,533.33

Plot Data for Monthly Order Cycle Model

Working Inventory Level = ...$ 139,533.33
Annual Number of Orders Cut = ... 53,880
Total Inventory Management Cost = ...$ 82,919.73

TABLE III-9A

The Same Inventory Model Shown in Table III-9
With Ordering Performed on an Economic Order Quantity Basis

Item Sales	Item Freq.	N	Total Orders	Order Quantity	Working Inventory
$ 100.00	1500	2.0	2965	$ 50.60	$ 37,947.33
$ 144.00	1000	2.4	2372	$ 60.72	$ 30,357.87
$ 289.00	700	3.4	2352	$ 86.01	$ 30,104.88
$ 441.00	500	4.2	2075	$ 106.25	$ 26,563.13
$ 900.00	380	5.9	2253	$ 151.70	$ 28,839.97
$ 1,600.00	250	7.9	1976	$ 202.39	$ 25,298.22
$ 4,900.00	100	13.8	1383	$ 354.18	$ 17,708.75
$10,000.00	50	19.8	988	$ 505.96	$ 12,649.11
$90,000.00	10	59.3	593	$ 1,517.89	$ 7,589.47
Totals	4490		16958		$217,058.73

Plot Data for Economic Order Quantity Inventory Model

Working Inventory Level = ...$217,058.73
Annual Number of Orders Cut = ... 16,958
Total Inventory Management Cost = ...$ 43,411.75

Program consists of a simple set of instructions, in the form of keypunched cards, to be followed by the computer in processing information introduced as input data. This data, consisting of annual item sales, item frequency, order cost and carrying cost are read in as part of the "source deck". The computer then processes this data according to the instructions it has received and performs, in a matter of seconds, arithmetic calculations it might take an efficient operator several man-days to accomplish using a desk calculator.

The purpose of this particular computer program was not only to simulate the inventory model shown in Table 9, but to provide data (Table 9D) from which to construct an Optimal Policy Curve (Figure III-5), for visual reference. In addition, it was instructed to determine inventory levels, total number of orders processed annually, and inventory management costs for several possible order policies at 5% increments of inventory restraint, the partial print-out for which is reproduced in Tables 9A - 9C.

With the Optimal Policy Curve before him, plotted with detailed information from the print-out, the inventory manager has a wealth of information from which he can make an intelligent decision as to which order policy will be best for his operation. Use of the computer obviates the necessity of calibrating the curve for values of -K- since this task is accomplished internally.

A quick glance at the Optimal Policy Curve shows the inventory manager that his monthly order cycle is highly

TABLE III-9B

The Same Inventory Model as in Table III-9A Except that Ordering is Performed on Optimal Policy Basis Under Restraint Limiting Working Inventory to 100.% of Original Inv. Level.

Item Sales	Item Freq.	N	Total Orders	Order Quantity	Working Inventory
$ 100.00	1500	3.1	4612	$ 32.53	$ 24,393.94
$ 144.00	1000	3.7	3689	$ 39.03	$ 19,515.15
$ 289.00	700	5.2	3659	$ 55.29	$ 19,352.52
$ 441.00	500	6.5	3228	$ 68.30	$ 18,075.76
$ 900.00	380	9.2	3505	$ 97.58	$ 18,539.39
$ 1,600.00	250	12.3	3072	$ 130.10	$ 16,262.63
$ 4,900.00	100	21.5	2152	$ 227.68	$ 11,383.84
$10,000.00	50	30.7	1537	$ 325.25	$ 8,131.31
$90,000.00	10	92.2	922	$ 975.76	$ 4,878.79
Totals	4490		26380		$139,533.32

Plot Data - Optimal Policy Model with Restraint. Limiting Working Inventory to 100.% of Original Inv. Level.

Working Inventory Level = ...$ 139,533.32
Annual Number of Orders Cut = ... 26,380
Total Inventory Management Cost = ...$ 47,719.10

TABLE III-9C

The Same Inventory Model as in Table III-9A Except that Ordering is Performed on Optimal Policy Basis Under Restraint Limiting Working Inventory to 125.% of Original Inv. Level.

Item Sales	Item Freq.	N	Total Orders	Order Quantity	Working Inventory
$ 100.00	1500	2.5	3689	$ 40.66	$ 30,492.42
$ 144.00	1000	3.0	2952	$ 48.79	$ 24,393.94
$ 289.00	700	4.2	2927	$ 69.12	$ 24,190.66
$ 441.00	500	5.2	2583	$ 85.38	$ 21,344.70
$ 900.00	380	7.4	2804	$ 121.97	$ 23,174.24
$ 1,600.00	250	9.8	2460	$ 162.63	$ 20,328.28
$ 4,900.00	100	17.2	1722	$ 284.60	$ 14,229.80
$10,000.00	50	24.6	1230	$ 406.57	$ 10,164.14
$90,000.00	10	73.8	738	$1,219.70	$ 6,098.48
Totals	4490		21104		$174,416.66

Plot Data - Optimal Policy Model with Restraint. Limiting Working Inventory to 125.% of Original Inv. Level.

Working Inventory Level = ...$ 174,416.66
Annual Number of Orders Cut = ... 21,104
Total Inventory Management Cost = ...$ 44,454.28

TABLE III-9D

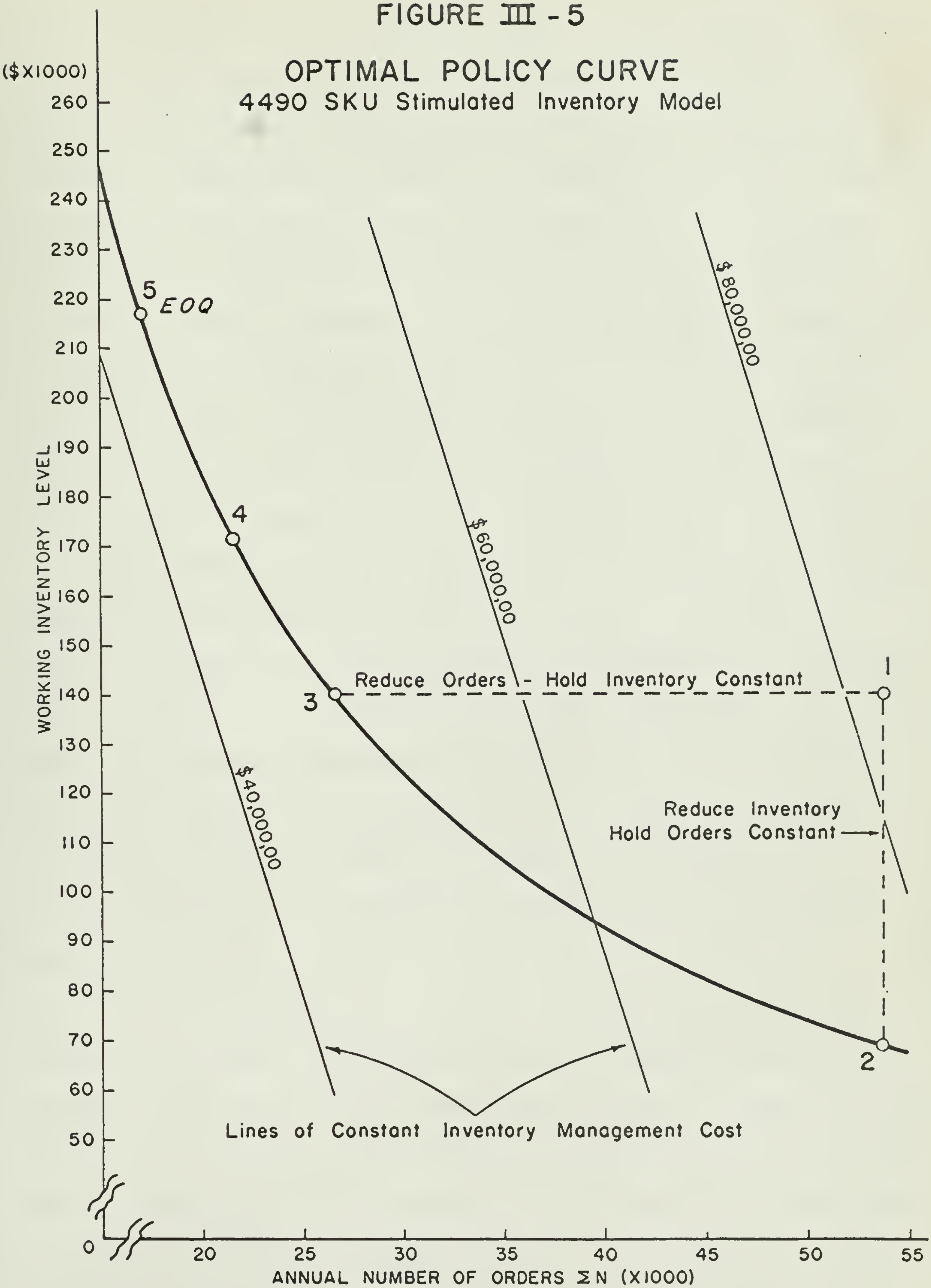
Data for Optimal Policy Curve Figure III-5
Includes Values of K for Calibration of O.P.C.

Working Inventory	Total Orders	K Value
$ 21450.00	171600.	0.5
$ 42900.00	85800.	1.0
$ 64350.00	57200.	1.5
$ 85800.00	42900.	2.0
$ 107250.00	34320.	2.5
$ 128700.00	28600.	3.0
$ 150150.00	24514.	3.5
$ 171600.00	21450.	4.0
$ 193050.00	19067.	4.5
$ 214500.00	17160.	5.0
$ 235950.00	15600.	5.5
$ 257400.00	14300.	6.0
$ 278850.00	13200.	6.5
$ 300300.00	12257.	7.0
$ 321750.00	11440.	7.5
$ 343200.00	10725.	8.0

FIGURE III - 5

OPTIMAL POLICY CURVE

4490 SKU Stimulated Inventory Model



inefficient (Pt. "1" on Figure III-5). Interpolating visually, he sees that total annual variable costs are approximately $83,000.00. Inventory can be cut by 50% and costs reduced to $75,000 by operating at point "2", with no change in numbers of orders processed annually. This would produce a useful Optimal Order Policy if the warehouse is badly overloaded. On the other hand, by holding inventory at a constant level, i.e. by imposing an inventory restraint of 100%, and operating at point "3", annual orders processed will be sharply reduced - from 53,880 to 26,380. With orders costing $1.28 each to process, this change alone results in a savings of $35,200.00.

The ultimate goal of course, is cost optimization represented by the EOQ Order Policy at point "5", with total annual management costs of $43,400.00. However, in the model under consideration, switching to an EOQ order basis would increase the working inventory level to $217,000.00, or 55.8%. Should this level of inventory prove too much to handle, an intermediate level at point "4" might prove acceptable. An inventory restraint of 125% would be utilized to accomplish this goal.

The data provided in Tables III-9B and 9C, extracted from the print-out, provides information regarding new order quantities by items should the manager decide to hold the line at original inventory level (100% restraint) or increase the level of restraint in proportion to any readily available unused warehouse spaces he might have at his disposal. Table III-9C provides order quantity information, by item, on an

Optimal Policy order basis. Ordering in these quantities will result in minimizing total inventory management costs under an inventory restraint of 125%. In similar manner, any practical restraint level could have been programmed.

In Chapter IV, an Optimal Policy Curve will be developed for an actual Navy Exchange Operation, and an investigation made into the applicability of this valuable decision-making technique to the Navy Exchange Program.

CHAPTER IV

A FLEXIBLE ORDER POLICY FOR NAVY EXCHANGES

Background

In October 1963, an instruction[45] issued by the Navy Ship's Store Office on the subject of adoption of the EOQ order policy for staple item merchandise was partially implemented. Complete compliance with this instruction by 30 June 1964 was directed.[46] Until implementation of the EOQ order policy, Navy Exchanges were placing orders for staple items essentially on a monthly order cycle. The purpose of the instruction was to introduce Navy Exchanges to a scientific inventory management technique which would materially reduce total variable costs of inventory management, while simultaneously lowering working inventory levels throughout the entire system. An immediate, and unexpected, reaction from many Exchanges came in the form of requests to raise inventory level allowances. The author was assigned to the Navy Ship's Store Office for temporary duty during the months of June and July 1964, and it was at this time that he became familiar with the apparent inventory problems encountered by some Exchanges following adoption of the EOQ order policy.

It seemed logical, at first, to assume that working inventory level would decline when an Exchange shifted to an EOQ order policy from basically a monthly order cycle. Very high volume items with annual sales in excess of $30,000.00 would be ordered on a weekly, rather than monthly, order

basis thereby decreasing the working inventory level by approximately 75% for those items.[47] Other items with annual sales ranging from $7,200.00 to $30,000.00 would be ordered bi/weekly rather than monthly with a consequent reduction of approximately 50% in working inventory level. Conversely, in the case of lower volume sales items, order quantities in terms of number of month's stock would increase in inverse proportion to an item's sales volume, to a maximum of six month's stock for items with annual sales less than $120.00. However, it was felt "intuitively" that the reduction of inventory levels of the high volume sales items would more than compensate for the increased inventories of low volume items, so that the net effect would be a decrease in total working inventory level. It was the intent of the author to investigate this problem by means of a computer simulated inventory model, fully expecting that the results would substantiate the Navy Ship's Store Office contention that lower total working inventory levels would be attained through adoption of an EOQ order policy.

The inventory models appearing in Tables 9 and 9A of Chapter III were the product of this simulation. However, the models did not behave as expected. Working inventory level resulting from ordering on an EOQ basis increased 55.8% over that produced by a monthly order cycle. The key to this unexpected phenomena lay in the assignment of item frequencies designed to produce a typical item mix by sales volume of a very large Group VI Navy Exchange. The frequencies assigned

were heavily weighted at the lower sales range. For example, items with annual sales of $144.00 were considered to be 150 times more numerous than items with annual sales of $90,000.00. Although such an assumed item mix would be reasonable in a civilian department store operation[48], it could not be considered as completely realistic in a Navy Exchange which carries a much narrower range of merchandise, most of which consists of fast moving high sales volume items.

Before any valid conclusion could be reached regarding the impact of an EOQ order policy on Exchange inventory levels, an inventory model would have to be constructed using staple item sales from an actual operation. Through the cooperation of the Navy Exchange Officer and Merchandise Manager at the Navy Exchange, Naval Amphibious Base, Little Creek, Virginia, actual sales data were obtained for every fifth staple item carried on their stock cards. These sales data were then utilized in the computer program developed in Appendix C, which in turn provides the basis for the investigation performed throughout this chapter.

The results, once again, were contrary to expectations. Whereas a working inventory level of $119,937.44 could be expected on a monthly order cycle for the 3245 SKU inventory, the inventory level increased to $210,383.66 on an EOQ order basis with assigned order and inventory carrying costs of $1.28 and 0.1 respectively. These order and inventory carrying costs have been determined by the Navy Ship's Store Office as being representative costs incurred within the Navy Exchange

program.[49] However, total inventory management costs decreased from $61,836.94 per annum on a monthly order cycle to $42,076.73 on an EOQ basis, thus realizing a significant annual saving. Therein lies the dilemma which must be faced by the Exchange inventory managers - the Navy Exchange Officer and Merchandise Manager.

An EOQ order policy reduces total inventory management costs but, in the inventory model analyzed, a problem has been created in coping with an anticipated increase in inventory level if the adequacy of warehouse space is marginal. A partial solution to this dilemma, in the form of a more flexible order policy - the Optimal Policy - is offered as a possible alternate course of action in situations such as this. In the remainder of this chapter, an Optimal Order Policy will be developed for the Navy Exchange, NAB, Little Creek, Va., and its general applicability to the Navy Exchange Program investigated.

Assistance In Decision-Making

Initial input to the computer program in Appendix A consisted of order and inventory carrying costs of $1.28 per item per order and 10% of working inventory level respectively. 649 annual item sales data values, representing 3245 stock keeping units, and a multiplication factor of 5 completed the input data. Since sales data had been taken from every fifth staple item stock card, the multiplication factor of 5 brought working inventory levels and management costs in line with

those which would actually be incurred in the Exchange operation.

Output from this program provided the figures from which the Optimal Policy Curve shown in Figure IV-1 was constructed. Point "1" on this curve represents the $\sum N - \sum INV$ relationship occurring on a monthly order cycle. This is a non-optimal orders - inventory relationship, as attested by the fact that point "1" does not fall on the Optimal Policy Curve. Data from which to plot this point were also part of the program output, as were all other data utilized throughout this chapter. The EOQ $\sum N - \sum INV$ relationship is indicated by point "5". As expected, this point falls on the curve since it represents an optimum relationship between numbers of orders processed annually and working inventory level.

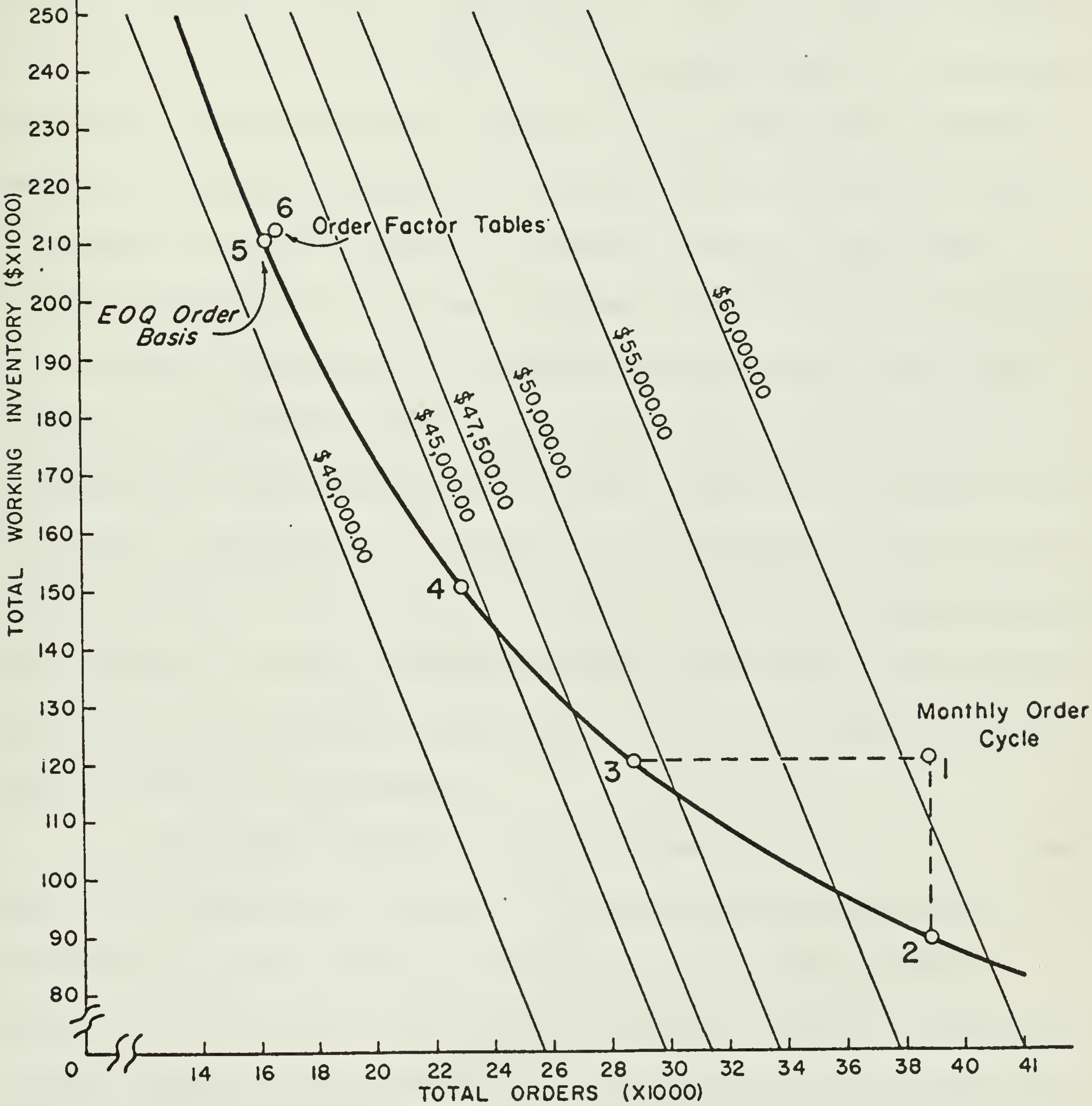
Using the assigned order and inventory carrying costs, lines of constant total inventory management costs are drawn to the curve starting at $40,000.00 and increasing in $5,000.00 increments to $60,000.00. A constant cost line at $47,500.00 is also indicated. Visual interpolation quickly places the annual variable management costs at approximately $62,000.00 and $42,000.00 for ordering on a monthly cycle and an EOQ order policy respectively. This observation is borne out by additional information on the computer print-out giving exact values of $61,836.94 and $42,076.73.

Once the Optimal Policy Curve is fully developed and the inventory models representing various order policies

FIGURE IV - I

OPTIMAL POLICY CURVE

Navy Exchange, NAB Little Creek, Va.



plotted as illustrated in Figure IV-1, the inventory manager has an extremely valuable decision-making tool at his disposal. He can see at a glance what action must be taken to improve the situation regarding management costs. For the case in point, several possibilities exist. If the working inventory level at $119,937.44 is over-taxing the warehouse capacity, dropping down to point "2" on the curve would result in reducing inventory to a level of $88,799.32, without changing the total number of orders processed per year. A simple modification of the program in Appendix A introducing an inventory restraint of 74.2% of working inventory maintained on a monthly order cycle, will provide a print-out of applicable order quantities (q_i) for each staple item carried in stock. Ordering in the quantities indicated will result in a reduction of inventory to the desired level. Inspection of the Optimal Policy Curve indicates a reduction in management costs of approximately $3,000.00 as an added bonus, if this Optimal Policy is pursued.

An Optimal Policy leading to point "3" on the curve results in an annual savings of approximately $13,000.00, verified by the computer print-out as being $12,939.56. This is 65.5% of the maximum savings realized through adoption of an EOQ order policy, but at no increase in working inventory level, i.e., under an imposed inventory restraint of 100% of the monthly order cycle inventory level. Finally, if an inventory restraint of 125% of the monthly order cycle level is

practical, annual savings would increase to $17,321.85, or 87.7% of maximum attainable.

Changing to an EOQ order basis, from a monthly order cycle, would produce the maximum savings of $19,760.21, but in so doing, the working inventory level would jump 75.5% to $210,383.66.

Utilization of the Order Factor Tables published in the Stock Control Handbook results in a modified EOQ order policy. It is a non-optimal order policy without limiting restraints on numbers of orders processed or inventory level. Although in this example, savings of $19,432.30 resulting from ordering out of the tables approach the maximum attainable, the increase in working inventory to a level of $211,387.29 poses an additional problem for the inventory manager. A recapitulation of the management costs and imputed savings, as well as inventory levels attained through adoption of various order policy alternatives is shown in Table IV-1.

It is quite apparent that the Optimal Policy Curve is a valuable scientific inventory management device for use by an experienced inventory manager. It shows him exactly how orders and inventory investment can be traded one for the other to achieve a desired goal. Used in coordination with a high speed electronic computer, utilizing a program such as shown in Appendix A, it provides a powerful assist to the decision-making process. The Optimal Order Policy provides an exceptionally flexible response to the many pressures which influence any inventory management decision.

TABLE IV-1

RECAPITULATION OF MANAGEMENT COSTS, WORKING INVENTORY LEVELS, AND IMPUTED SAVINGS RESULTING FROM VARIOUS ORDER POLICIES.

Inventory Models for Staple Item Merchandise of Navy Exchange, NAB Little Creek, Va.

Assumptions: $C_o = \$1.28$; $I = .1$ NOTE: Pts. "1", "2" etc. refer to Fig. IV-1

Order Policy	Working Inv. Level ($\sum$INV)	Orders Processed Annually ($\sum$N)	Total Annual Inv.Mgt.Costs	Annual Savings	% Maximum Savings
Monthly Order Cycle (Pt."1")	$119,937.44	38,940	$61,836.94	-0-	-
Optimal Policy (Pt."2") Inv. Restraint - 74.2% of monthly order cycle level	$ 88,799.32	38,940	$58,723.13	$ 3,113.81	15.75
Optimal Policy (Pt. "3") Inv. Restraint - 100.0%	$119,937.44	28,831	$48,897.38	$12,939.56	65.6
Optimal Policy (Pt."4") Inv. Restraint - 125.0%	$149,921.82	23,065	$44,515.09	$17,321.85	87.7
EOQ Order Basis (Pt."6")	$210,383.66	16,436	$42,076.73	$19,760.21	100.0
Modified EOQ Basis (Pt. "5") Order Factor Tables	$211,387.29	16,614	$42,404.65	$19,432.30	98.3

Note: The author fully appreciates that in a real application of the technique described in this section, and elsewhere within this thesis, computed costs, savings, and inventory levels would be rounded off to the nearest $1,000.00 for an operation of the size described. However, exact figures have been used so that the reader might more readily compare them to their source in the computer print-out.

Erroneous Order and Carrying Costs.

Up to this point, the values of order and carrying cost of $1.28 and 0.1 ascribed to Navy Exchange Operations by the Navy Ship's Store Office have been accepted as correct. However, as pointed out by Starr and Miller[50], Buchan and Koenigsberg[51], and others, it is extremely difficult to determine these costs accurately. In the case of Navy Exchanges located in 23 states including Alaska and Hawaii, as well as in many foreign countries, there will be a considerable variation in such factors as wage scales, markdowns, stock shrinkage and self-insurance losses, all of which have a marked effect on imputed costs. At best then, the costs assigned as being applicable to all Exchanges must be average costs for the Navy Exchange Program as a single entity. Costs applicable to a particular Exchange could only be determined from an extensive analysis conducted at that Exchange, not on the basis of a systemwide average.

To determine the effect of order and carrying cost variations on EOQ working inventory levels and total inventory

management costs under various order policies, fifty combinations of C_o and I were introduced as input data into the computer program in Appendix A. A portion of the output is tabulated in Table IV-2.

At certain foreign Exchanges where wage scales are low, but adverse factors contributing to carrying costs tend to make them higher than average, order and carrying costs of $1.00 and 0.15 respectively would be more representative of real costs than the system average. An examination of management costs, annual savings, and inventory levels attributed to these cost values proves extremely interesting. An optimal order policy, with an inventory restraint of 100% of that resulting from a monthly order cycle, will achieve annual savings of 88.8% of maximum attainable while holding the line on inventory at the $119,937.44 level. On the other hand, ordering by Order Factor Tables results in a savings of only 75.6% of maximum attainable, in spite of the fact that the inventory level soars to $211,387.29. Once again, the inventory manager needs only a quick glance at the Optimal Policy Curve Figure IV-2 to discern this fact and decide on the best course of action to follow. The decision as to which order policy to follow would be elementary since, in this case, pursuing an Optimal Order Policy (Pt. "2") results in both larger savings and a lower inventory level than ordering from the Order Factor Tables (Pt. "4"). Ordering on an Optimal Policy basis under an imposed inventory restraint of 125% would,in this case, also result in

TABLE IV-2

COMPARISON OF INVENTORY MANAGEMENT COSTS RESULTING
FROM USE OF VARIOUS ORDER POLICIES

CSTORD	CSTINV	SUMINV	EOQINV	OFTINV	CSTEOQ	CSTMOC	CSTOPC	PCTOPC	CSTOFT	PCTOFT
$	$	$	$	$	$	$	$	%	$	%
1.00	.05	119,937.	262,980.	211,387.	26,298.	44,937.	34,828.	54.2	27,183.	95.2
1.00	.10	119,937.	185,955.	211,387.	37,191.	50,934.	40,825.	73.6	37,753.	95.9
1.00	.134	119,937.	160,941.	211,387.	42,971.	54,952.	44,843.	84.4	44,834.	84.4
1.00	.15	119,937.	151,831.	211,387.	45,549.	56,931.	46,822.	88.8	48,322.	75.6
1.00	.20	119,937.	131,490.	211,387.	52,596.	62,928.	52,818.	97.8	58,891.	39.1
1.28	.05	119,927.	297,527.	211,387.	29,752.	55,840.	42,901.	49.6	31,835.	92.0
1.28	.10	119,937.	210,384.	211,387.	42,077.	61,837.	48,897.	65.5	42,404.	98.3
1.28	.15	119,937.	171,778.	211,387.	51,533.	67,834.	54,894.	79.4	52,974.	91.2
1.28	.170	119.937.	160,865.	211,387.	54,920.	70,242.	57,302.	84.4	57,320.	84.3
1.28	.20	119,937.	148,764.	211,387.	59,505.	73,831.	60,891.	90.3	63,543.	71.8
1.28	.25	119,937.	133,058.	211,387.	66,529.	79,828.	66,888.	97.3	74,113.	43.0
1.35	.10	119,937.	216,060.	211,387.	43,212.	64,562.	50,916.	63.9	43,567.	98.3
1.35	.15	119,937.	176,412.	211,387.	52,924.	70,559.	56,912.	77.4	54,137.	93.1
1.35	.181	119,937.	160,596.	211,387.	58,136.	74,278.	60,630.	84.4	60,690.	84.3
1.35	.20	119,937.	152,777.	211,387.	61,111.	76,556.	62,909.	88.4	64,706.	76.7
1.65	.05	119,937.	337,803.	211,387.	33,780.	70,248.	53,568.	45.7	37,982.	88.5
1.65	.10	119,937.	238,863.	211,387.	47,772.	76,245.	59,565.	58.6	48,552.	97.3
1.65	.15	119,937.	195,031.	211,387.	58,509.	82,242.	65,562.	70.3	59,121.	97.4
1.65	.20	119,937.	168.902.	211,387.	67,561.	88,238.	71,559.	80.7	69,690.	89.7

Explanation of Column headings: (Note. These are variables used in computer program.)

CSTORD - Order cost per item per order.
CSTINV - Inventory carrying cost as a percentage of inventory level per annum.
SUMINV - Working inventory level - monthly order cycle order basis.
EOQINV - Working inventory level - EOQ order basis.
OFTINV - Working inventory level when order quantities are determined by Order Factor Tables
CSTEOQ - Total variable management cost - EOQ order basis
CSTMOC - Total variable management cost - Monthly order cycle

TABLE IV-2 continued

CSTOPC - Total variable management cost - Optimal Policy basis under restraint limiting working inventory level to 100% of that resulting from Monthly order cycle.

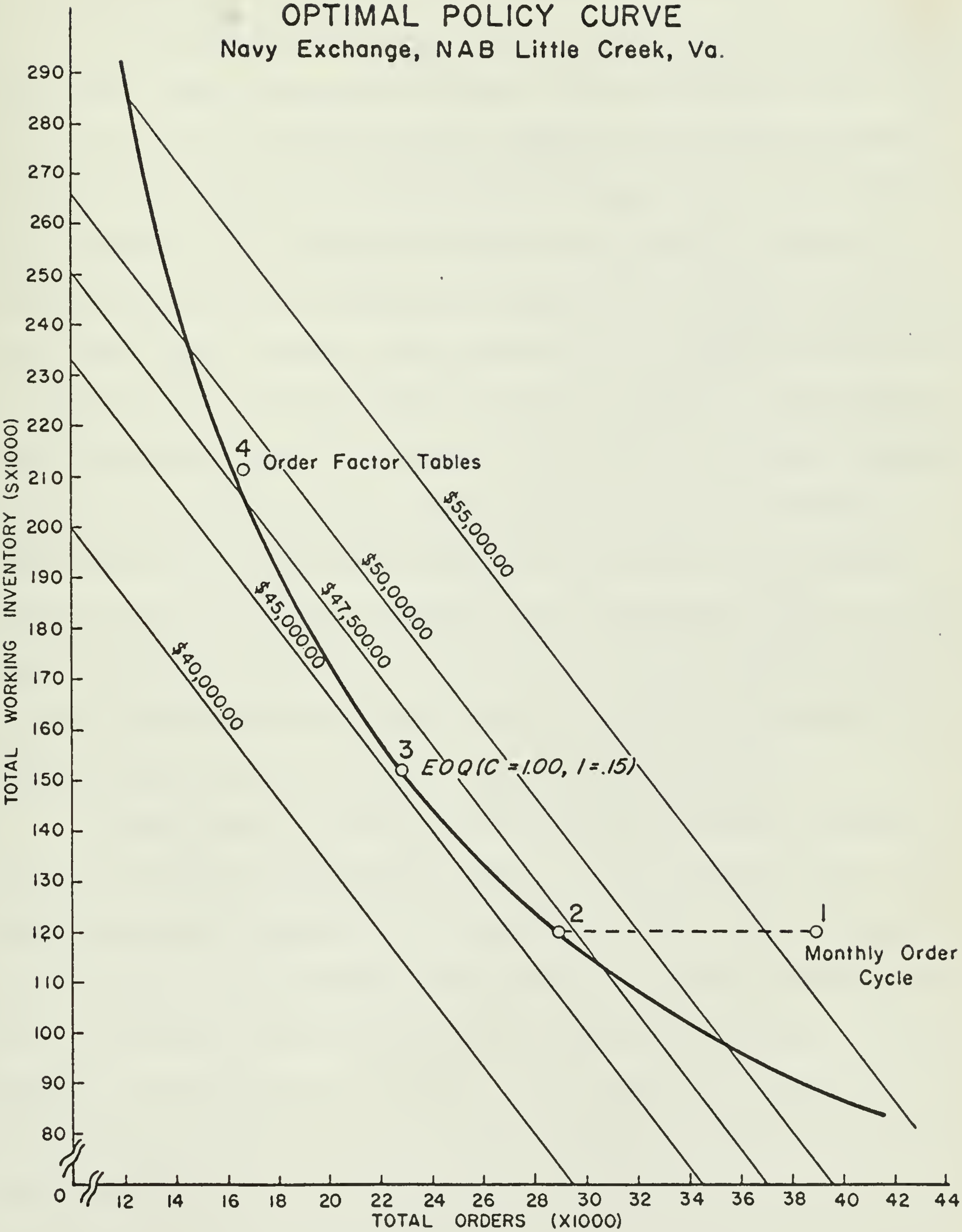
PCTOPC - Percent of maximum possible savings in utilizing the Optimal Policy compared to savings through EOQ order policy.

CSTOFT - Total variable management cost - when utilizing Order Factor Tables.

PCTOFT - Percent of maximum possible savings in ordering by Order Factor Tables compared to savings through EOQ order policy.

FIGURE IV-2

OPTIMAL POLICY CURVE
Navy Exchange, NAB Little Creek, Va.



achieving maximum savings since the resulting inventory level is the same as for an EOQ order basis (Pt. "3").

A more detailed inspection of Table IV-2 discloses the fact that when the ratio of order cost to carrying cost ($\frac{C_o}{I}$) is high, as for example at 20, ($\frac{1.00}{.05}$), the savings achieved through ordering on an Order Factor Table basis are considerably higher than realized through ordering on an Optimal Policy basis with an inventory restraint of 100%. Conversely, when this ratio is low, for example at 5, ($\frac{1.00}{.20}$), the savings realized on an Optimal Policy basis are greater. The break point where savings are equal is at a C_o : I ratio of approximately 7.5, as for example C_o of $1.28 and an I of 0.17. This observed phenomenon can be attributed to the varying slope of the lines of constant total management cost, as cost relationships between C_o and I vary. At all combinations of C_o and I for which the ratio of C_o : I is 7.5, the EOQ inventory level would be very close to $160,900.00.

Once again, a simple mathematical analysis explains why the inventory management costs are the same for the two order policies represented by points 2 and 4. If a tangent line representing minimum total costs is drawn to the curve at an inventory level of $160,900, and perpendicular lines drawn to it from points 2 and 4, the lengths of these perpendiculars are found to be equal, regardless of what combination of C_o : I is represented, provided that their ratio is 7.5. Since a perpendicular distance between lines of constant total cost represents an increment of cost,

perpendiculars of equal length would represent equal cost increments. These increments in turn represent a degree of inefficiency in comparison to the minimum or EOQ cost resulting from an EOQ order policy.

The point made in the two preceding paragraphs is a bit obscure. The observed phenomenon of equal costs, resulting from the two order policies being compared when the $C_o : I$ cost ratio is 7.5, was mentioned here by way of drawing attention to the fact that as actual values of C_o and I vary, so does the cost effectiveness of an order policy based upon arbitrarily assumed order and carrying costs. In the particular model being analysed, ordering from the order factor tables always resulted in higher inventory levels than did ordering on an optimal policy basis. This drawback was offset by lower total costs. However, if actual costs were such that a ratio of $\frac{C_o}{I} = 7.5$ existed, then ordering on an optimal policy basis produced not only a lower inventory level but lower total costs as well. Cost data tabulated in Table IV-2 substantiates this contention. For a cost ratio of 6.7 $(\frac{1.00}{.15})$, total inventory management costs on an optimal policy basis (100% inventory restraint) would amount to $46,822. while ordering on an order factor table basis would result in costs of $48,322.

CHAPTER V

SUMMARY AND RECOMMENDATIONS

In this thesis, an attempt has been made to investigate the highly productive functional area of Scientific Inventory Management, where its tools, devices and techniques specifically relate to application in Navy Exchange operations. The simple mathematical treatment accorded more commonly used SIM formulas and concepts, was intended to fill a void which the author feels exists between procedures to be followed, as outlined in the Stock Control Handbook, and a thorough understanding of the mathematical rationale underlying the concepts involved in mechanical application of these procedures. Even those managers who take a dedicated approach toward their duties - and the percentage is high - tend to apply new concepts mechanically rather than coming to grips with principles involved. Once these principles are thoroughly understood, efficient application follows naturally.

Through adoption of scientific techniques, the frequency of stock-outs and of overstocking is reduced to an acceptable minimum, and the expense of handling inventory is likewise minimized. However, blind application of the tools of scientific inventory management will not guarantee management efficiency. Maximum utility of SIM formulas is realized only through their intelligent application by personnel fully aware of all facets of a particular inventory problem. They

must be equally aware of the pitfalls to be encountered, as well as the benefits to be derived, through use of such powerful techniques as the Economic Order Quantity Concept. Both the advantages and disadvantages resulting from application of an EOQ ordering policy were illustrated in Chapter III.

In the author's research on the subject of EOQ, he encountered the device known as the Optimal Policy Curve, which was developed in Chapter III. It provides an inventory manager with the means to best utilize his knowledge and experience to trade orders and inventory investment, one for the other, in order to optimize his own unique operation. In Chapter IV, this device was investigated by application of its technique to a real inventory situation with information provided by the Navy Exchange, United States Naval Amphibious Base, Little Creek, Va.

The results of this investigation indicate that an Optimal Order Policy, would provide individual Exchanges with a more flexible order policy than is now in effect. Such an order policy would be a logical extension of presently employed SIM techniques, making full use of the powerful advantages offered by computer applications.

Its goal would be minimization of total inventory management costs under imposed restraints, dictated by the inherent limitations of a particular operation. In view of the benefits to be derived from development of a

workable Optimal Order Policy, the author believes that additional study of this powerful scientific management technique is merited.

APPENDICES

APPENDIX A

COMPUTER PROGRAM TO DEVELOP DATA FROM WHICH TO CONSTRUCT AN OPTIMAL ORDER POLICY CURVE

Purpose

The purpose of this program is to compute order numbers-inventory level relationship data from which to construct an Optimal Order Policy Curve for the Navy Exchange operation, NAB, Little Creek, Va. In addition, the program computes order quantities, total numbers of orders processed annually, working inventory levels, and total inventory management costs for various order policies (1) Monthly order cycle (2) EOQ order basis (3) Modified EOQ order policy utilizing "order factor tables" and (4) Optimal Order Policy under varying levels of inventory restraint.

Language

Fortran II (IBM 7040 Computer).

Symbolic Dictionary

Variable	S/A*	I/O**	Description
CSTORD	S	I	Order cost.
CSTINV	S	I	Inventory carrying cost
FREQ	S	I	Multiplication factor of 5 to compensate for using only every fifth sales value from stock cards.
SALES	A	I	Annual sales per item.

---

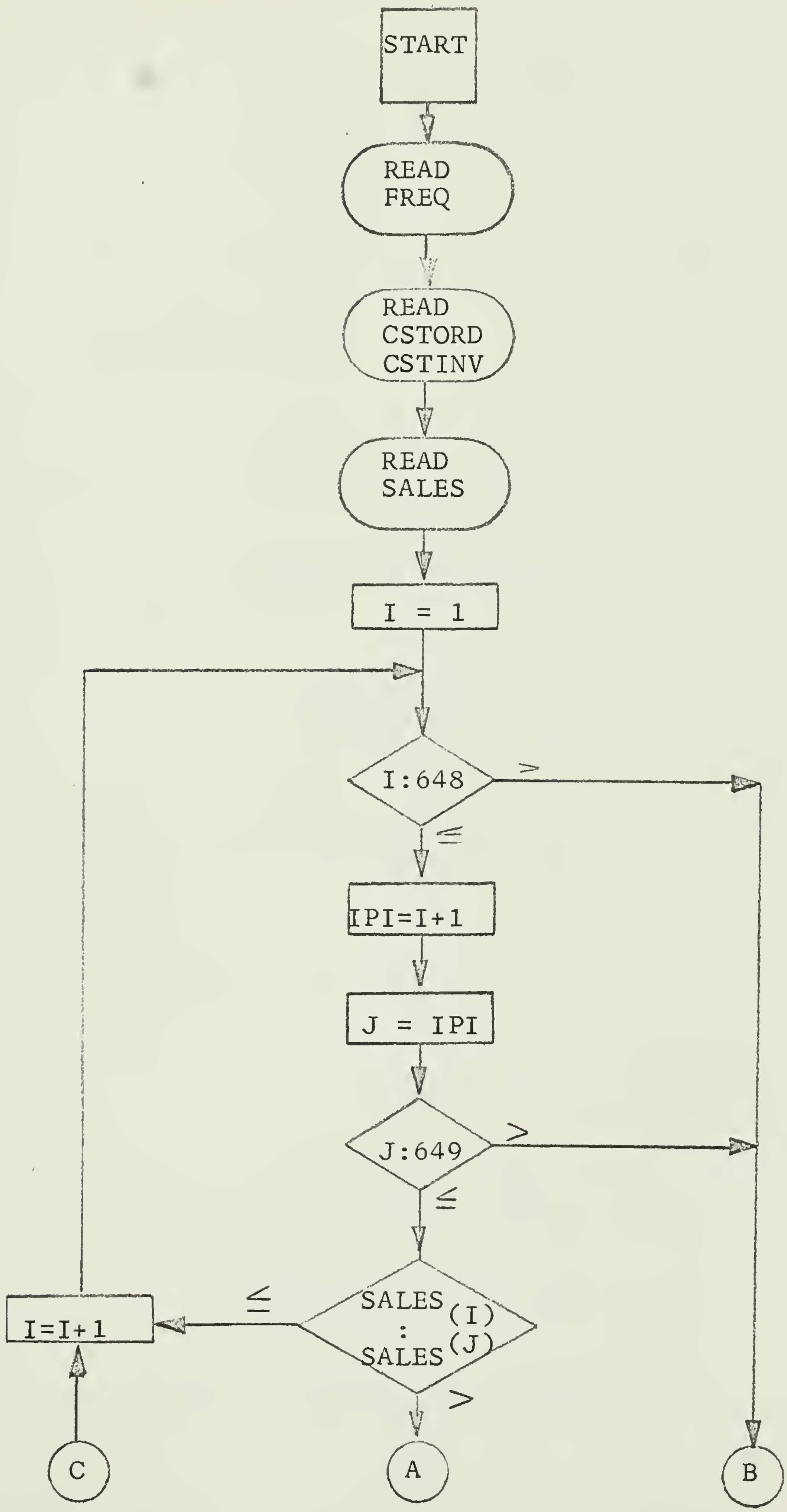
*S - Single variable; A - Array of variables
**I - Input; O - Output

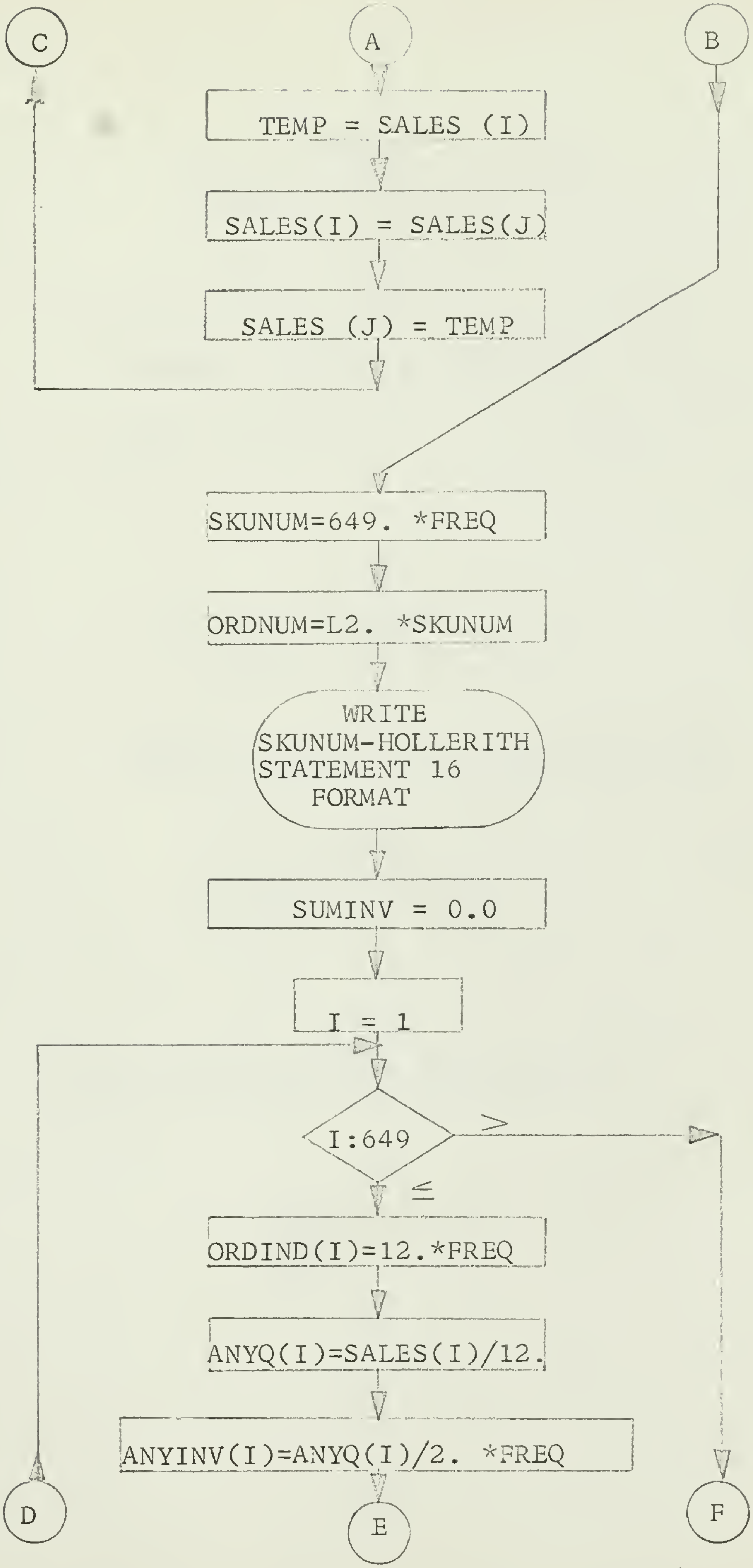
Variable	S/A	I/O	Description
TEMP	S	---	A holding variable utilized in arranging sales data in ascending order of annual volume.
SKUNUM	S	0	Total number of SKU's
ORDNUM	S	0	Total number orders processed Annually-Monthly order cycle.
ORDIND	A	0	Annual number of orders processed per item.
ANYQ	A	0	Item order quantity
ANYINV	A	0	Item working inventory level
SUMINV	S	0	Total working inventory level - Monthly order cycle.
CSTMOC	S	0	Total inventory management cost - Monthly order cycle.
EOQ	A	0	Item order quantity - EOQ order basis.
SUMEOQ	S	0	Total working inventory level - EOQ order basis.
EOQN	A	0	Number of orders processed annually per item - EOQ order basis.
SMEOQN	S	0	Total number of orders processed annually - EOQ order basis.
EOQINV	S	0	Total working inventory level - EOQ order basis.
CSTEOQ	S	0	Total inventory management costs - EOQ order basis.
DIFF	S	0	Cost savings EOQ order policy compared to monthly-order cycles basis.
RATIO	S	---	Internal variable used to compare EOQ inventory level with that of a monthly order cycle.
A	S	0	Internal variable - same purpose as above.
SUMSRS	S	---	Sums of square roots of item sales.

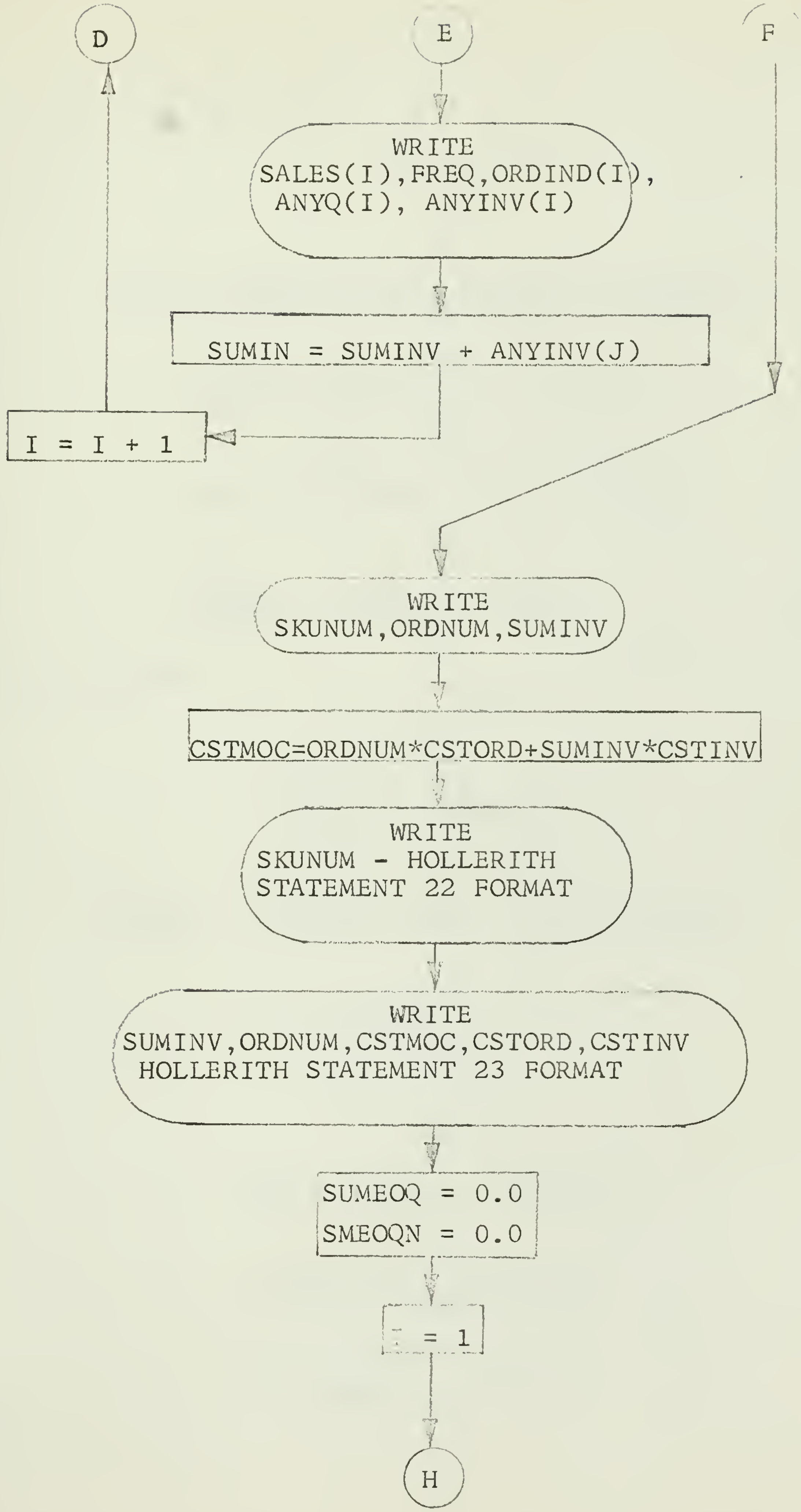
Variable	S/A	I/O	Description
PROD	S	---	Product of $\sum N \times \sum INV$
VALK	S	0	Value of constant -K-
PLTINV	S	0	Inventory level - half of ordered pair in Optimal Policy Curve plot.
PLTORD	S	0	Number of orders processed - second half of ordered pair in OPC plot.
AA	S	---	Incremental internal variable - starts at 1 and adds one through each pass of DO loop.
B	S	---	Incremental internal variable - each pass of loop adds 5% to level of inventory restraint.
WRKINV	S	0	Optimal policy working inventory level at various incremental restraints.
C	S	0	Inventory restraint in terms of percent of monthly order cycle level.
ORDERS	S	0	Total number of orders processed annually for various inventory restraints on optimal policy basis.
TOTINV	S	0	Total working inventory level on optimal policy basis.
CSTOPC	S	0	Total inventory management cost for a particular level of restraint.
CSTDIF	S	0	Cost savings - optimal order policy compared to monthly order cycle.
PCTOPC	S	0	Percentage of maximum savings - optimal policy compared to EOQ basis.
OFTQ	A (if desired)	0	Item order quantity - order factor tables order basis.
OFTINV	A (if desired)	0	Item working inventory level - order factor tables order basis.
OFTORD	A (if desired)	0	Annual number of orders cost per item - order factor tables basis.

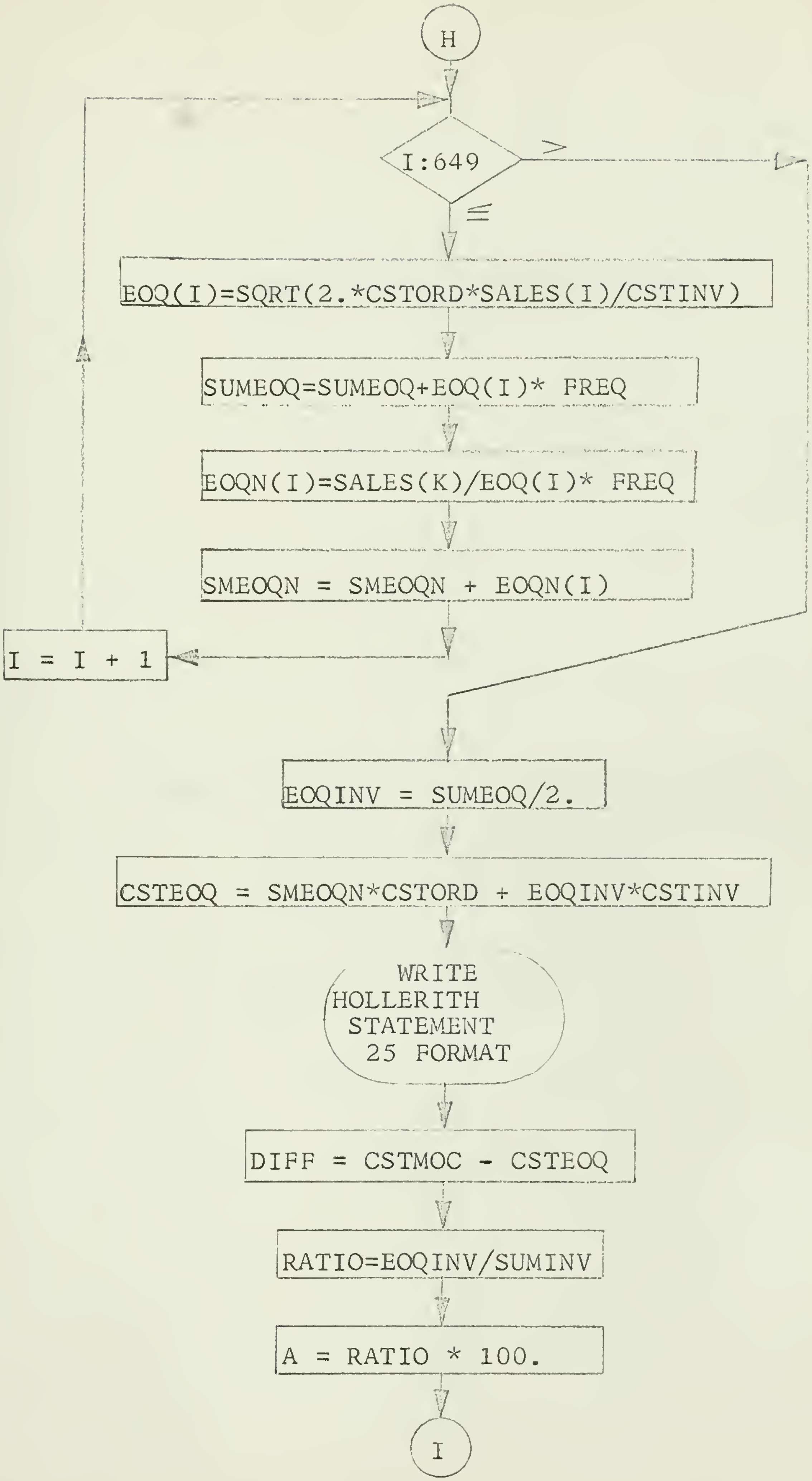
Variable	S/A	I/O	Description
SIMINV	S	0	Total working inventory level - order factor tables basis.
SIMORD	S	0	Total number of orders processed annually - order factor table basis.
CSTOFT	S	0	Total inventory management cost - order factor tables basis.
DIFOFT	S	0	Annual savings - order factor tables basis compared to monthly order cycle.
PCTOFT	S	0	Percentage of maximum possible savings order factor tables basis compared to EOQ order basis.

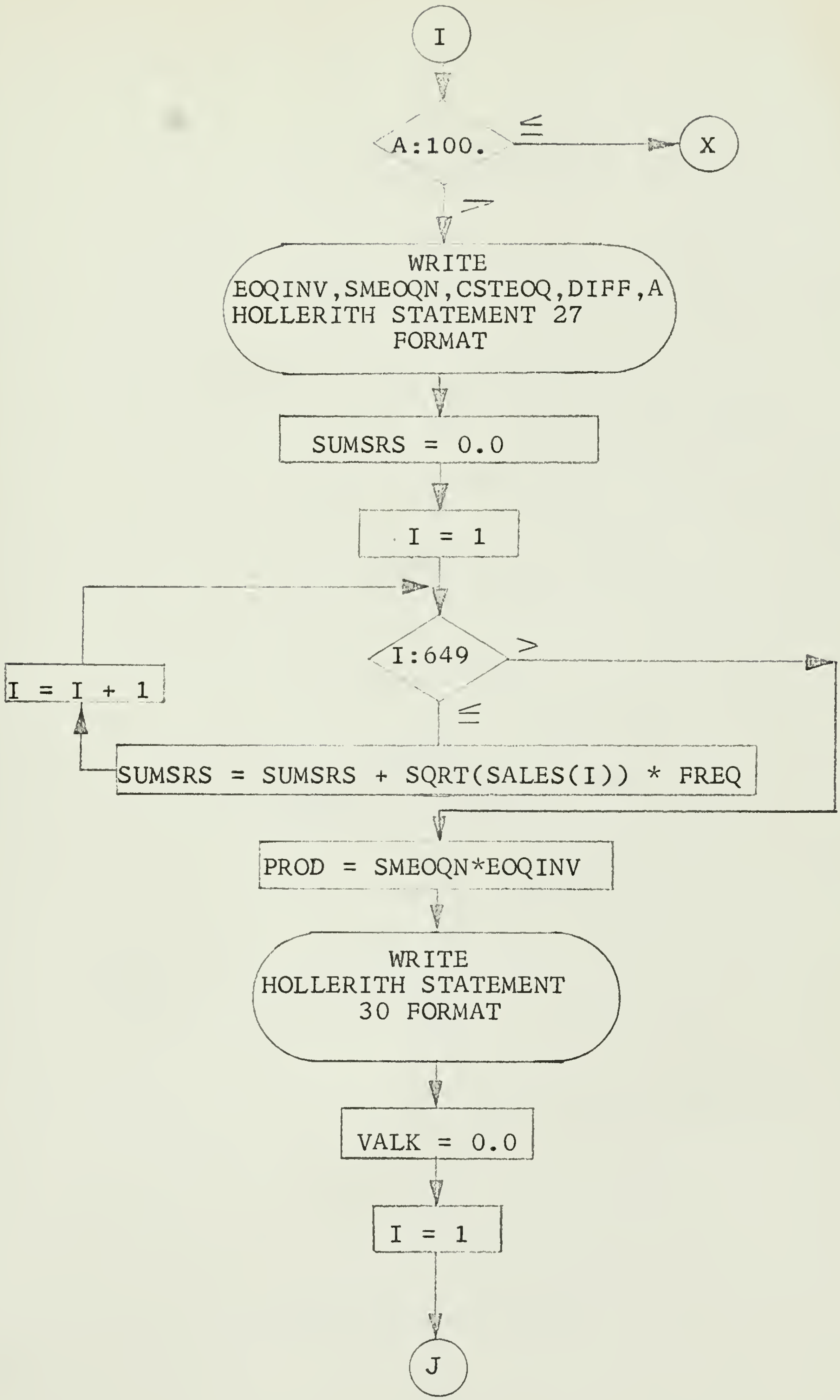
FLOW DIAGRAM

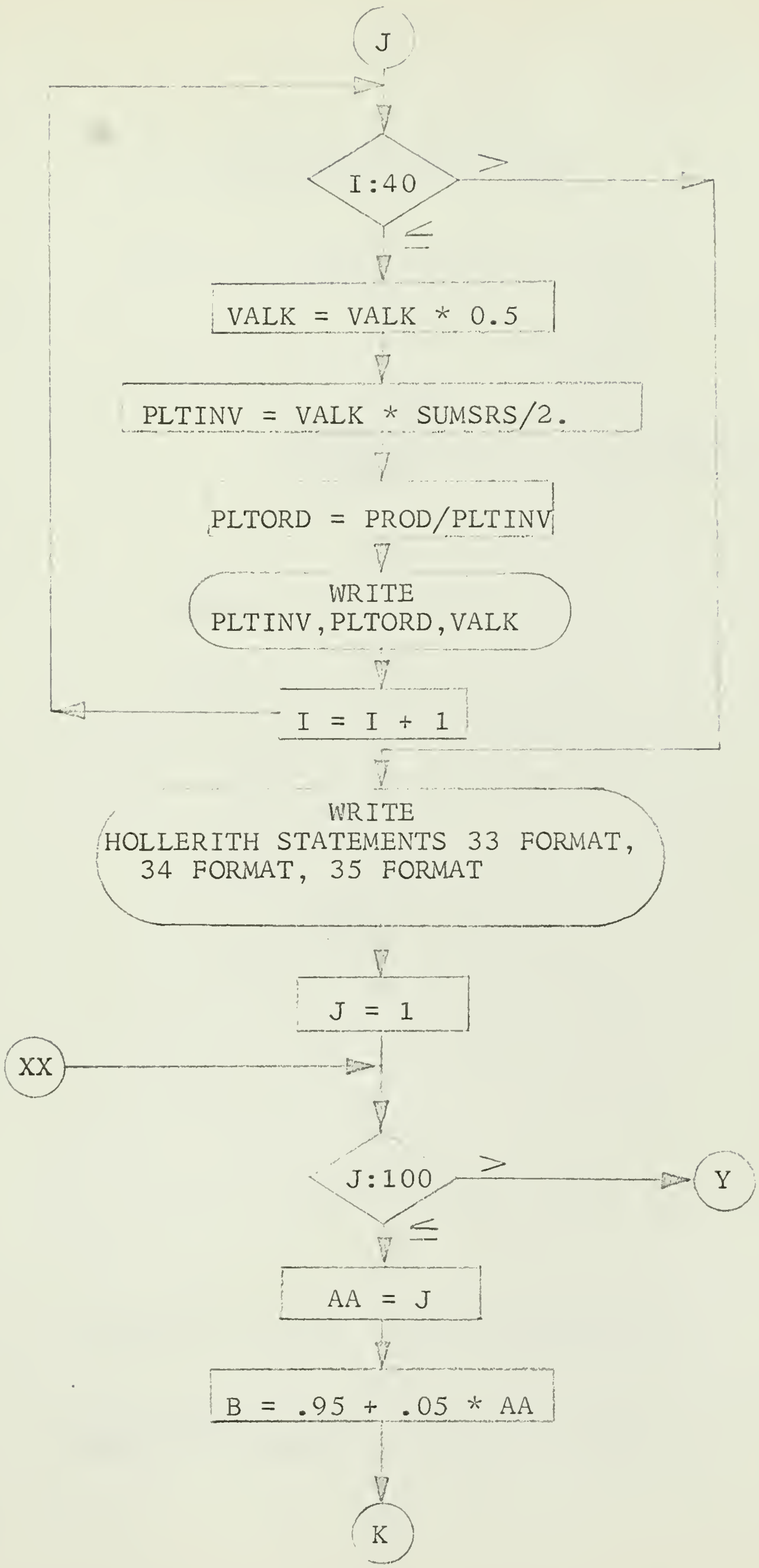


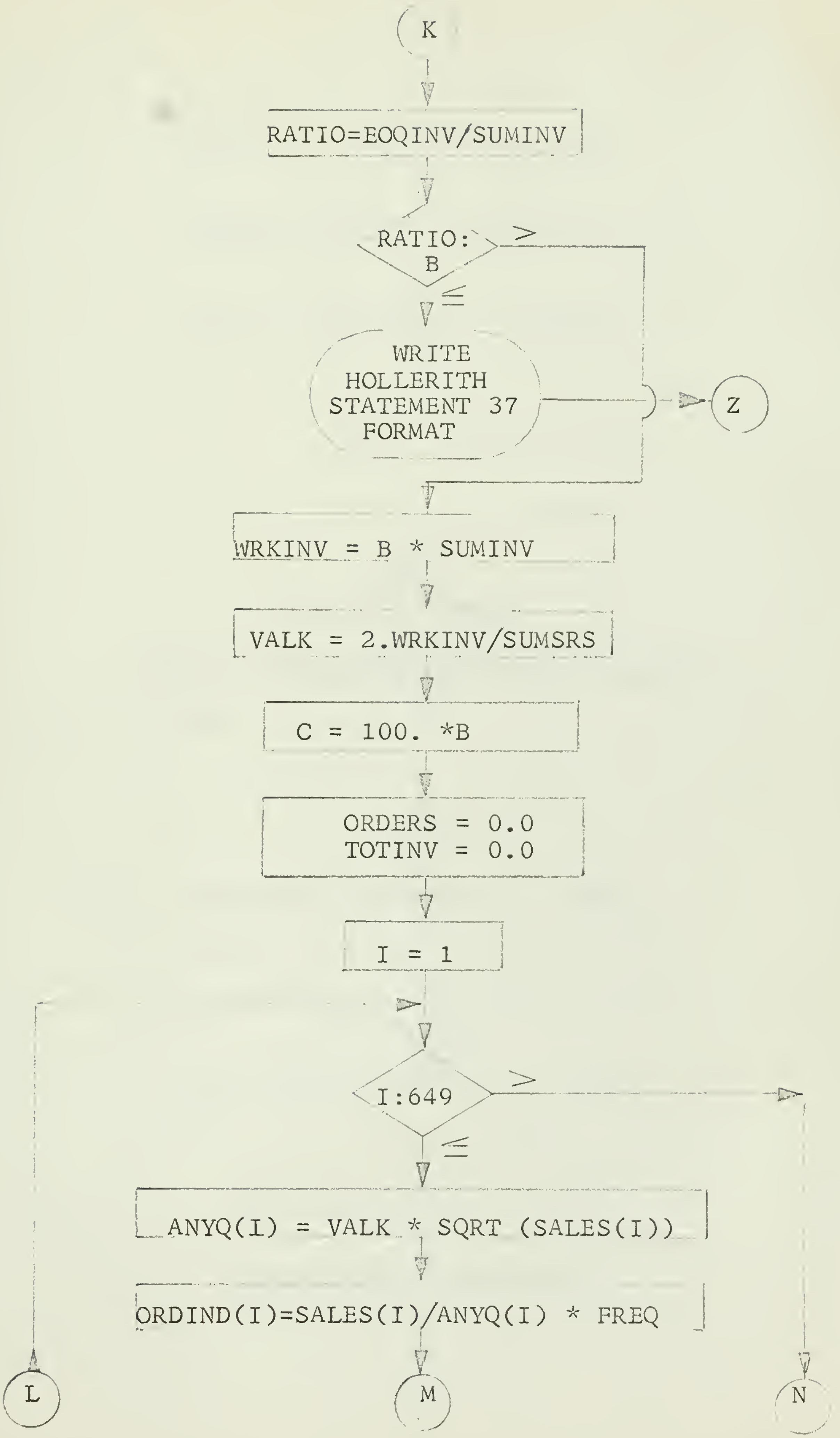

C
A
B
TEMP = SALES (I)
SALES(I) = SALES(J)
SALES (J) = TEMP
SKUNUM=649. *FREQ
ORDNUM=L2. *SKUNUM
WRITE
SKUNUM-HOLLERITH
STATEMENT 16
FORMAT
SUMINV = 0.0
I = 1
I:649
>
≤
ORDIND(I)=12.*FREQ
ANYQ(I)=SALES(I)/12.
ANYINV(I)=ANYQ(I)/2. *FREQ
D
E
F

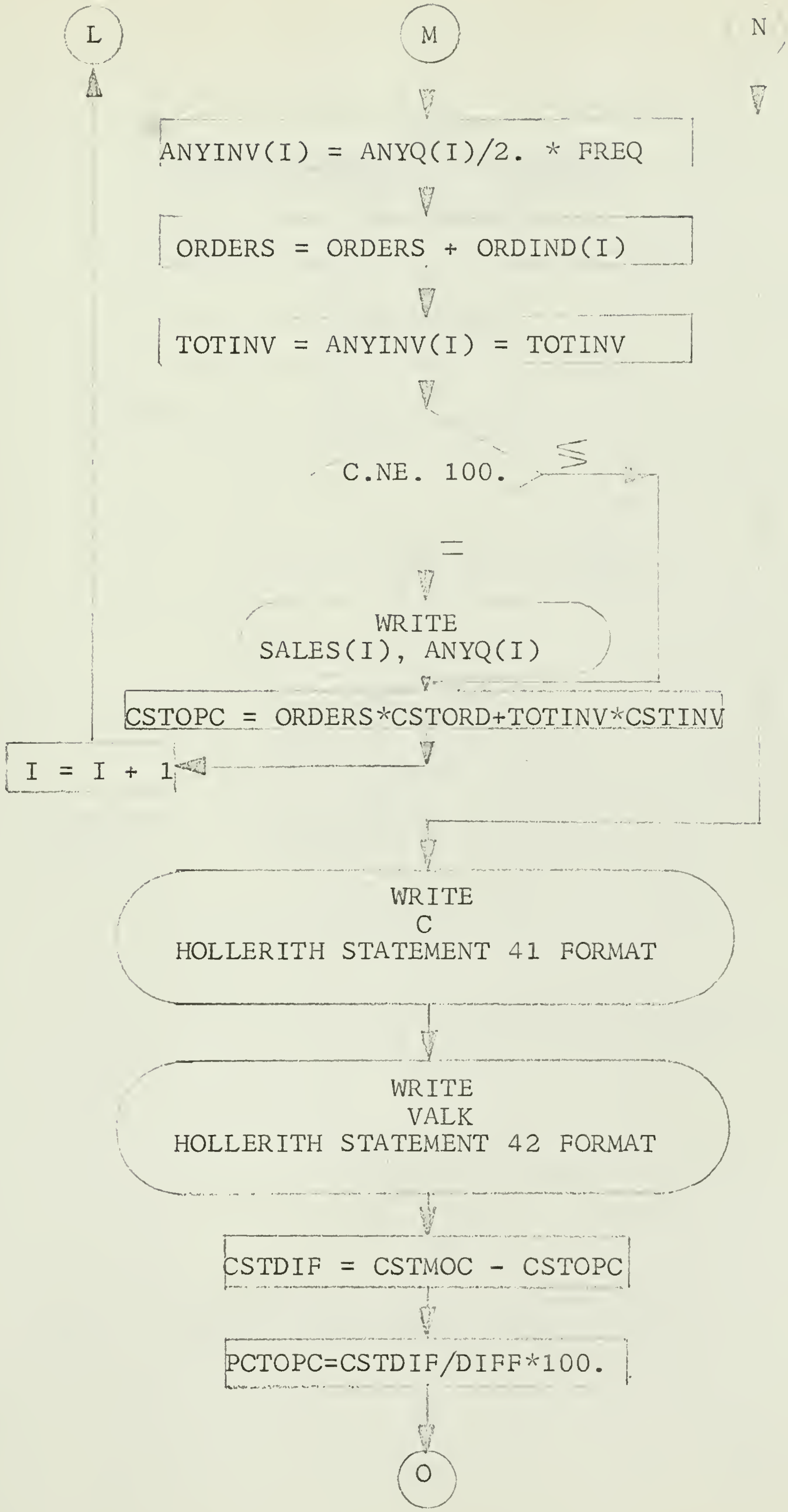

D
E
F
WRITE
SALES(I),FREQ,ORDIND(I),
ANYQ(I), ANYINV(I)
SUMIN = SUMINV + ANYINV(J)
I = I + 1
WRITE
SKUNUM,ORDNUM,SUMINV
CSTMOC=ORDNUM*CSTORD+SUMINV*CSTINV
WRITE
SKUNUM - HOLLERITH
STATEMENT 22 FORMAT
WRITE
SUMINV,ORDNUM,CSTMOC,CSTORD,CSTINV
HOLLERITH STATEMENT 23 FORMAT
SUMEOQ = 0.0
SMEOQN = 0.0
= 1
H

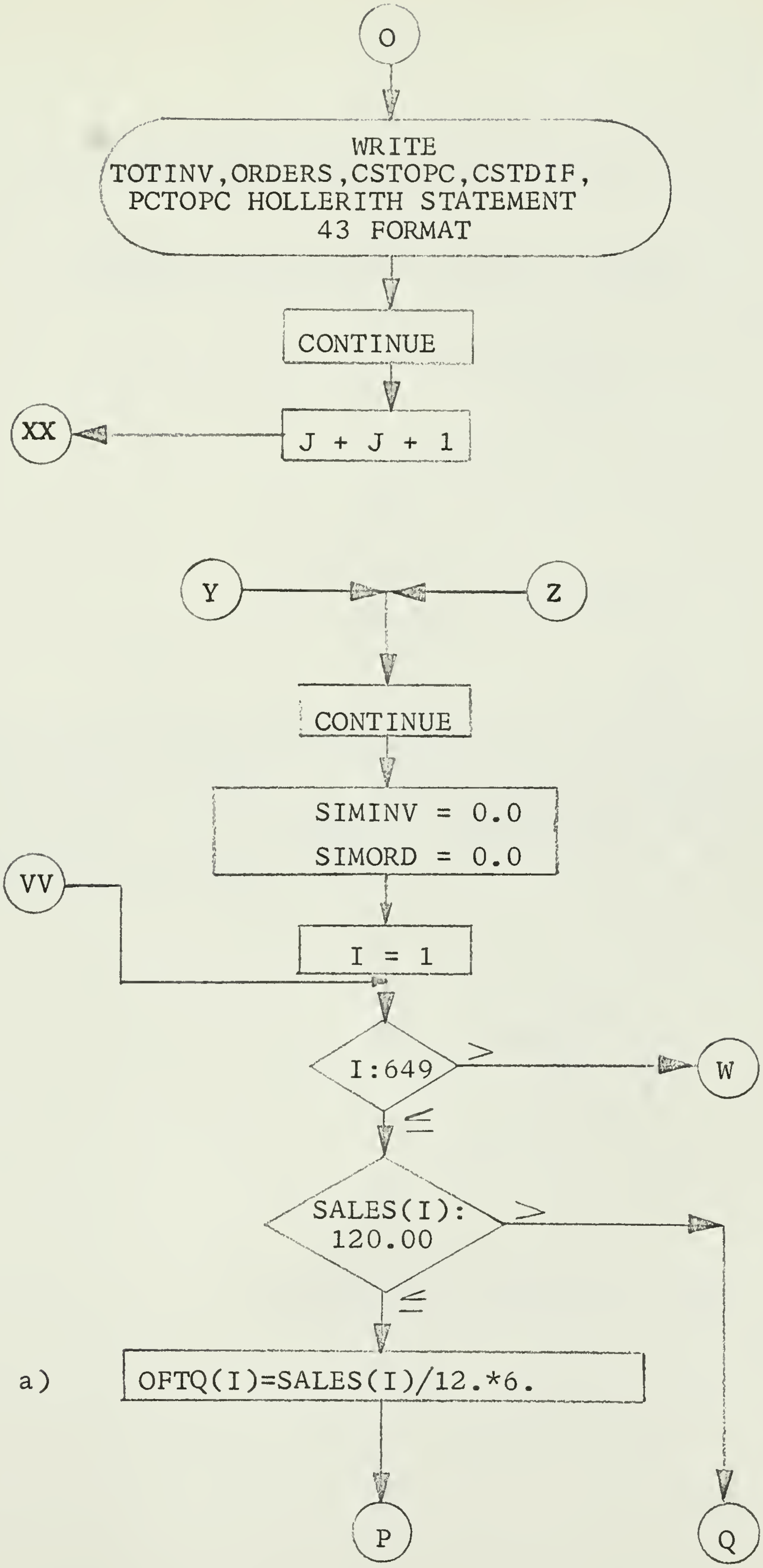

H
I:649
>
≤
EOQ(I)=SQRT(2.*CSTORD*SALES(I)/CSTINV)
SUMEOQ=SUMEOQ+EOQ(I)* FREQ
EOQN(I)=SALES(K)/EOQ(I)* FREQ
SMEOQN = SMEOQN + EOQN(I)
I = I + 1
EOQINV = SUMEOQ/2.
CSTEOQ = SMEOQN*CSTORD + EOQINV*CSTINV
WRITE HOLLERITH STATEMENT 25 FORMAT
DIFF = CSTMOC - CSTEOQ
RATIO=EOQINV/SUMINV
A = RATIO * 100.
I

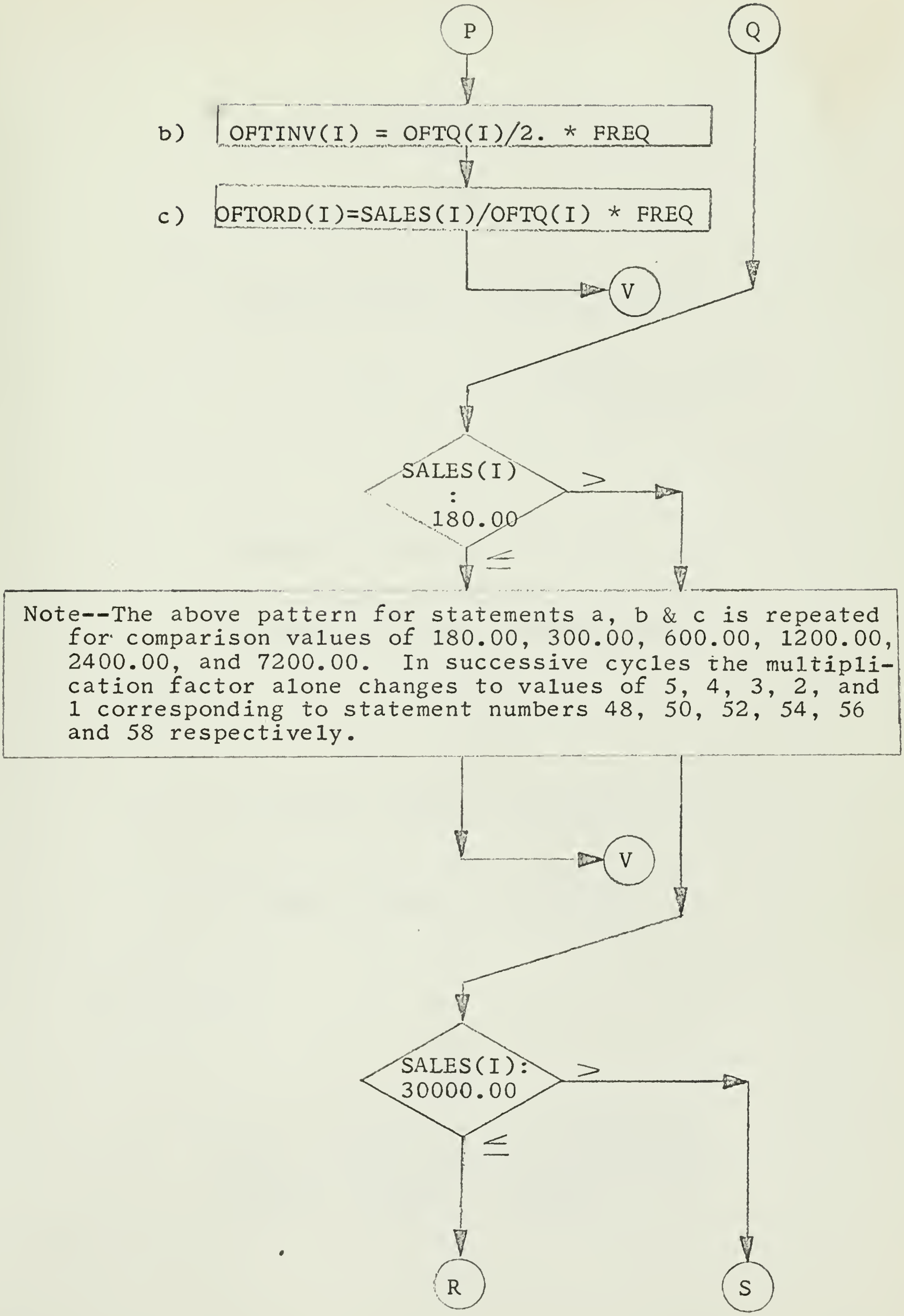

I
A:100.
≤
X
>
WRITE
EOQINV,SMEOQN,CSTEOQ,DIFF,A
HOLLERITH STATEMENT 27
FORMAT
SUMSRS = 0.0
I = 1
I:649
>
≤
I = I + 1
SUMSRS = SUMSRS + SQRT(SALES(I)) * FREQ
PROD = SMEOQN*EOQINV
WRITE
HOLLERITH STATEMENT
30 FORMAT
VALK = 0.0
I = 1
J

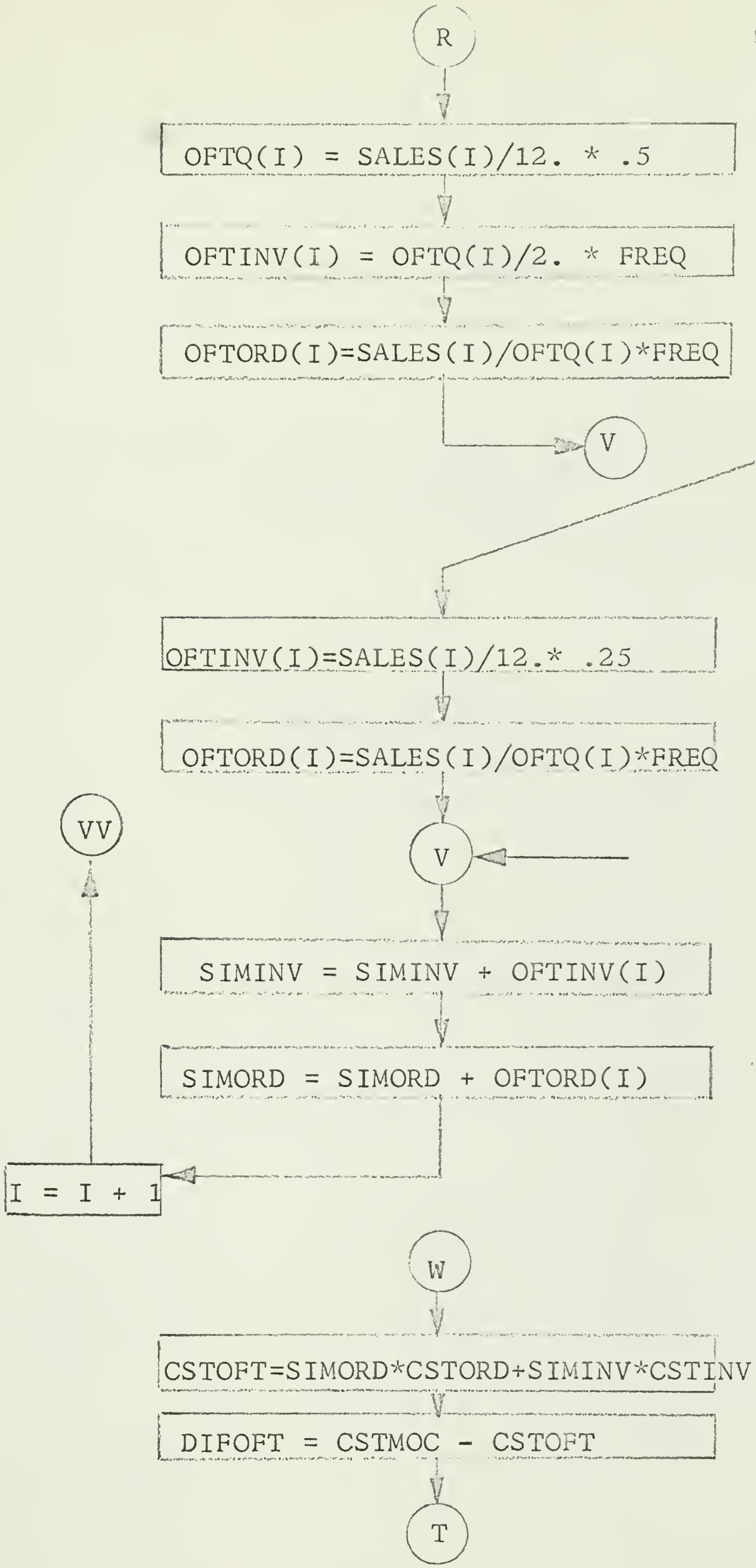

J
I:40
>
≤
VALK = VALK * 0.5
PLTINV = VALK * SUMSRS/2.
PLTORD = PROD/PLTINV
WRITE
PLTINV,PLTORD,VALK
I = I + 1
WRITE
HOLLERITH STATEMENTS 33 FORMAT,
34 FORMAT, 35 FORMAT
J = 1
XX
J:100
>
Y
≤
AA = J
B = .95 + .05 * AA
K

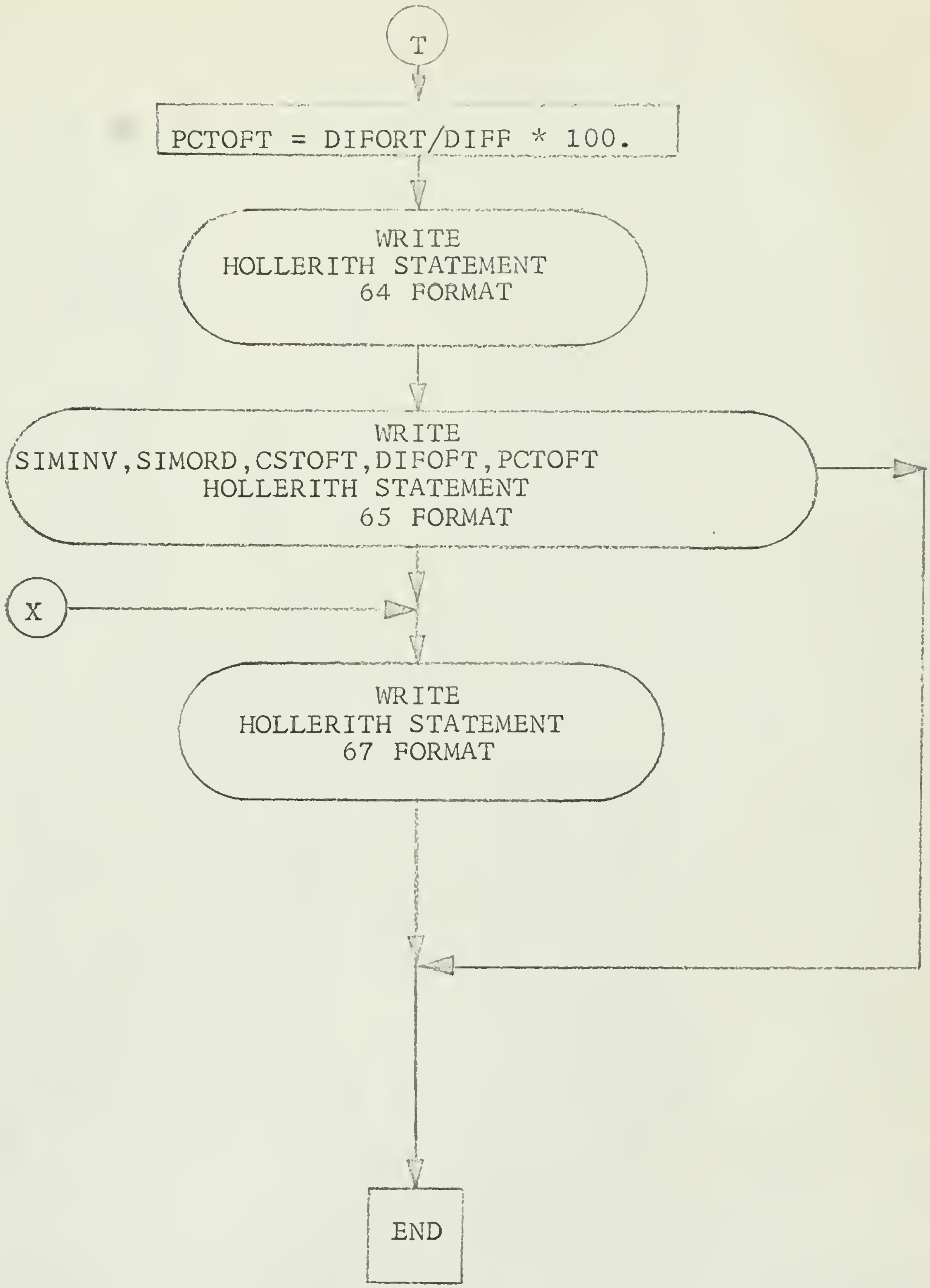

K
RATIO=EOQINV/SUMINV
RATIO:
B
>
≤
WRITE
HOLLERITH
STATEMENT 37
FORMAT
Z
WRKINV = B * SUMINV
VALK = 2.WRKINV/SUMSRS
C = 100. *B
ORDERS = 0.0
TOTINV = 0.0
I = 1
I:649
>
≤
ANYQ(I) = VALK * SQRT (SALES(I))
ORDIND(I)=SALES(I)/ANYQ(I) * FREQ
L
M
N


L
M
N
ANYINV(I) = ANYQ(I)/2. * FREQ
ORDERS = ORDERS + ORDIND(I)
TOTINV = ANYINV(I) = TOTINV
C.NE. 100.
≶
=
WRITE
SALES(I), ANYQ(I)
CSTOPC = ORDERS*CSTORD+TOTINV*CSTINV
I = I + 1
WRITE
C
HOLLERITH STATEMENT 41 FORMAT
WRITE
VALK
HOLLERITH STATEMENT 42 FORMAT
CSTDIF = CSTMOC - CSTOPC
PCTOPC=CSTDIF/DIFF*100.
O


O
WRITE
TOTINV,ORDERS,CSTOPC,CSTDIF,
PCTOPC HOLLERITH STATEMENT
43 FORMAT
CONTINUE
J + J + 1
XX
Y
Z
CONTINUE
SIMINV = 0.0
SIMORD = 0.0
VV
I = 1
I:649
>
W
≤
SALES(I):
120.00
>
≤
a)
OFTQ(I)=SALES(I)/12.*6.
P
Q


P
Q
b) OFTINV(I) = OFTQ(I)/2. * FREQ
c) OFTORD(I)=SALES(I)/OFTQ(I) * FREQ
V
SALES(I)
:
180.00
>
≤
Note--The above pattern for statements a, b & c is repeated for comparison values of 180.00, 300.00, 600.00, 1200.00, 2400.00, and 7200.00. In successive cycles the multiplication factor alone changes to values of 5, 4, 3, 2, and 1 corresponding to statement numbers 48, 50, 52, 54, 56 and 58 respectively.
V
SALES(I):
30000.00
>
≤
R
S


R
S
OFTQ(I) = SALES(I)/12. * .5
OFTINV(I) = OFTQ(I)/2. * FREQ
OFTORD(I)=SALES(I)/OFTQ(I)*FREQ
V
OFTINV(I)=SALES(I)/12.* .25
OFTORD(I)=SALES(I)/OFTQ(I)*FREQ
VV
V
SIMINV = SIMINV + OFTINV(I)
SIMORD = SIMORD + OFTORD(I)
I = I + 1
W
CSTOFT=SIMORD*CSTORD+SIMINV*CSTINV
DIFOFT = CSTMOC - CSTOFT
T


T
PCTOFT = DIFORT/DIFF * 100.
WRITE
HOLLERITH STATEMENT
64 FORMAT
WRITE
SIMINV,SIMORD,CSTOFT,DIFOFT,PCTOFT
HOLLERITH STATEMENT
65 FORMAT
X
WRITE
HOLLERITH STATEMENT
67 FORMAT
END

FORTRAN SOURCE LIST

SOURCE STATEMENT

```
C      - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C      COMPUTER PROGRAM DESIGNED TO PRODUCE ORDER QUANTITY DATA FROM WHICH TO
C      ORDER STAPLE WAREHOUSED MERCHANDISE ON AN OPTIMAL TOTAL VARIABLE INVENTORY
C      MANAGEMENT COST BASIS.  PROGRAM INPUT - ANNUAL SALES $ PER ITEM OF STOCK,
C      ORDER COST, AND INVENTORY CARRYING COST.  PROGRAM OUTPUT - INVENTORY
C      MANAGEMENT COST DATA RESULTING FROM USE OF THREE ORDERING POLICIES - (1).
C      MONTHLY ORDER CYCLE.  (2). ON AN EOQ BASIS UTILIZING THE ORDER FACTOR
C      TABLES IN THE NSSO STOCK CONTROL HANDBOOK.  (3). ON AN OPTIMAL BASIS UNDER
C      IMPOSED WORKING INVENTORY LEVEL RESTRAINTS.
C      - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
       DIMENSION SALES(649), EOQ(649), ANYINV(649), ANYQ(649)
       DIMENSION OFTINV(649), OFTQ(649), OFTORD(649)
       DIMENSION EOQN(649), ORDIND(649)
       READ (5,10) FREQ
    10 FORMAT (F3.0)
       READ (5,11) CSTORD, CSTINV
    11 FORMAT (2F10.0)
       READ (5,12) SALES
    12 FORMAT (13F6.0)
       DO 14 1  =  1,648
       IPI  =  1 + 1
       DO 14 J  =  IPI, 649
       IF (SALES(I) - SALES(J) 14,14,13
    13 TEMP  =  SALES(I)
       SALES(I)  =  SALES(J)
       SALES(J)  =  TEMP
    14 CONTINUE
       WRITE (6,15)
    15 FORMAT (//)
       WRITE (6,68)
       SKUNUM  =  649.  *FREQ
       ORDNUM  =  12. * SKUNUM
       WRITE (6,16) SKUNUM
    16 FORMAT (22X, 9H DATA FOR, F6.0, 28H SKU INVENTORY MODEL. ANNUAL/
```

```
      EOQINV  =  SUMEOQ/2.
      CSTEOQ  =  SMEOQN * CSTORD  +  EOQINV * CSTINV
      WRITE (6,25)
   25 FORMAT (18X, 44H PLOT DATA FOR INVENTORY MODEL WITH ORDERING/
     116X, 47H PERFORMED ON AN ECONOMIC ORDER QUANTITY BASIS./
     213X, 54H ACTUAL SALES DATA OBTAINED FROM NAVEXCH LITTLE CREEK./)
      DIFF  =  CSTMOC - CSTEOQ
      RATIO  =  EOQINV/SUMINV
      A  =  RATIO * 100.
      IF (A - 100.) 66,66,26
   26 WRITE (6,27) EOQINV, SMEOQN, CSTEOQ, DIFF, A
   27 FORMAT (15X, 40H WORKING INVENTORY LEVEL = ............$, F10.2/
     115X, 40H ANNUAL NUMBER OF ORDERS CUT = ........., F8.0/
     215X, 40H TOTAL INVENTORY MANAGEMENT COST = ....$, F10.2/
     315X, 40H ANNUAL SAVINGS REALIZED THROUGH EOQ = $, F10.2/
     415X, 45H EOQ WORKING INVENTORY LEVEL HAS INCREASED TO, F6.1/
     515X, 52H PERCENT OF LEVEL RESULTING FROM MONTHLY ORDER CYCLE//)
      WRITE (6,68)
      SUMSRS  =  0.0
      DO 28 I  =  1,649
   28 SUMSRS  =  SUMSRS + SQRT(SALES(I)) * FREQ
      PROD  =  SMEOQN * EOQINV
      WRITE (6,29)
   29 FORMAT (15X, 45H DATA FOR OPTIMAL POLICY CURVE PLOT INCLUDING/
     114X, 47H VALUES OF -K- FOR CALIBRATION OF O.P.C. CURVE./)
      WRITE (6,30)
   30 FORMAT (15X, 43H WORKING INVENTORY   TOTAL ORDERS   K VALUE)
      VALK  =  0.0
      DO 32 I  = 1,40
      VALK  =  VALK + 0.5
      PLTINV  =  VALK * SUMSRS/2.
      PLTORD  =  PROD/ PLTINV
      WRITE (6,31) PLTINV, PLTORD, VALK
   31 FORMAT (17X, 2H $, F11.2, 8X, F7.0,   7X, F5.1)
   32 CONTINUE
C     - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C     ASSUMING THAT THE WORKING INVENTORY OBTAINED THROUGH ORDERING IN ECONOMIC
```

```
C         ORDER QUANTITIES WILL BE CONSIDERABLY HIGHER THAN THAT EXPERIENCED WITH
C         THE MONTHLY ORDER CYCLE, THE REMAINDER OF THE PROGRAM WILL DETERMINE THE
C         OPTIMAL ORDER POLICY TO BE FOLLOWED UNDER FIXED RESTRAINTS IMPOSED ON INV-
C         ENTORY LEVEL, COMMENCING WITH A RESTRAINT OF 100 PERCENT OF MONTHLY ORDER
C         CYCLE INVENTORY LEVEL AND INCREASING IN 5 PERCENT INCREMENTS UNTIL THE EOQ
C         WORKING INVENTORY LEVEL HAS BEEN REACHED.
C         - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
          WRITE (6,68)
          WRITE (6,33)
       33 FORMAT (20X, 49H ORDER QUANTITIES FOR EACH ITEM OF INVENTORY WHEN/
         120X, 48H ORDERING PERFORMED ON OPTIMAL POLICY BASIS WITH/
         219X, 50H INVENTORY RESTRAINT FIXED AT 100 PERCENT OF LEVEL/
         321X, 47H CARRIED WHEN OPERATING ON MONTHLY ORDER CYCLE./)
          WRITE (6,34)
       34 FORMAT (20X, 23H ANNUAL SALES     ORDER)
          WRITE (6,35)
       35 FORMAT (22X, 23H PER ITEM      QUANTITY/)
          DO 44 J  =  1,100
          AA =  J
          B  =  .95  +  .05 * AA
          RATIO  =  EOQINV/SUMINV
          IF (RATIO - B) 36,36,38
       36 WRITE (6,37)
       37 FORMAT (15X, 65H INVENTORY RESTRAINT NOT EXCEEDED - PROCEED TO ORD
         1ER ON EOQ BASIS//)
          WRITE (6,68)
          GO TO 45
       38 WRKINV  =  B * SUMINV
          VALK  =  2. * WRKINV/SUMSRS
          C  =  100. * B
          ORDERS  =  0.0
          TOTINV  =  0.0
          DO 40 I  =  1,649
          ANYQ(I)  =  VALK * SQRT(SALES(I)
          ORDIND(I)  =  SALES(I)/ANYQ(I) * FREQ
          ANYINV(I)  =  ANYQ(I)/2. * FREQ
          ORDERS  =  ORDERS + ORDIND(I)
```

```
      TOTINV  =  ANYINV(I)  +  TOTINV
      IF (C. NE. 100.) GO TO 40
      WRITE (6,39) SALES(I), ANYQ(I)
   39 FORMAT (20X, 2H $, F9.2, 5X, 2H $, F7.2)
   40 CSTOPC  =  ORDERS * CSTORD + TOTINV * CSTINV
      WRITE (6,15)
      WRITE (6,41) C
   41 FORMAT (21X, 46H PLOT DATA FOR OPTIMAL POLICY ORDER BASIS WITH/
     118X,41H RESTRAINT LIMITING WORKING INVENTORY TO ,F4.0,8H PERCENT/
     217X, 56H OF LEVEL CARRIED WHEN OPERATING ON MONTHLY ORDER CYCLE./
     317X, 56H ACTUAL SALES DATA OBTAINED FROM NAVEXCH LITTLE CREEK VA/)
      WRITE (6,42) VALK
   42 FORMAT (20X, 44H VALUE OF CONSTANT -K- FOR THIS INVENTORY = ,F6.3)
      CSTDIF  =  CSTMOC - CSTOPC
      PCTOPC  =  CSTDIF/DIFF * 100.
      WRITE (6,43) TOTINV, ORDERS, CSTOPC, CSTDIF, PCTOPC
   43 FORMAT (20X, 40H WORKING INVENTORY LEVEL = ............$, F10.2/
     120X, 40H ANNUAL NUMBER OF ORDERS CUT = ........., F8.0/
     220X, 40H TOTAL INVENTORY MANAGEMENT COST = ....$, F10.2/
     320X, 40H ANNUAL SAVINGS THROUGH OPTIMAL POLICY=$, F10.2/
     420X, 40H PERCENTAGE OF MAX. POSSIBLE SAVINGS = ., F9.1/
     515X, 54H * * * * * * * * * * * * * * * * * * * * * * * * * * * *//)
   44 CONTINUE
   45 CONTINUE
      SIMINV  =  0.0
      SIMORD  =  0.0
      DO 63 I  =  1,649
      IF (SALES(I) - 120.00) 46,46,47
   46 OFTQ(I) = SALES(I)/12. * 6.
      OFTINV(I)  =  OFTQ(I)/2. * FREQ
      OFTORD(I)  =  SALES(I)/OFTQ(I) * FREQ
      GO TO 62
   47 IF (SALES(I) - 180.00) 48,48,49
   48 OFTQ(I) = SALES(I)/12. *5.
      OFTINV(I)  =  OFTQ(I)/2. * FREQ
      OFTORD(I)  =  SALES(I)/OFTQ(I) * FREQ
      GO TO 62
```

```
49 IF (SALES(I) - 300.00) 50,50,51
50 OFTQ(I) = SALES(I)/12. *4.
   OFTINV(I)  =  OFTQ(I)/2. * FREQ
   OFTORD(I)  =  SALES(I)/OFTQ(I) * FREQ
   TO TO 62
51 IF (SALES(I) - 600.00) 52,52,53
52 OFTQ(I) = SALES(I)/12. *3.
   OFTINV(I)  =  OFTQ(I)/2. * FREQ
   OFTORD(I)  =  SALES(I)/OFTQ(I) * FREQ
   GO TO 62
53 IF (SALES(I) - 1200.00) 54,54,55
54 OFTQ(I) = SALES(I)/12. *2.
   OFTINV(I)  =  OFTQ(I)/2. * FREQ
   OFTORD(I)  =  SALES(I)/OFTQ(I) * FREQ
   GO TO 62
55 IF (SALES(I) - 2400.00) 56,56,57
56 OFTQ(I) = SALES(I)/12. *1.5
   OFTINV(I)  =  OFTQ(I)/2. * FREQ
   OFTORD(I)  =  SALES(I)/OFTQ(I) * FREQ
   GO TO 62
57 IF (SALES(I) - 7200.00) 58,58,59
58 OFTQ(I) = SALES(I)/12. *1.
   OFTINV(I)  =  OFTQ(I)/2. * FREQ
   OFTORD(I)  =  SALES(I)/OFTQ(I) * FREQ
   GO TO 62
59 IF (SALES(I) - 30000.00) 60,60,61
60 OFTQ(I)  =  SALES(I)/12. * .5
   OFTINV(I)  =  OFTQ(I)/2. * FREQ
   OFTORD(I)  =  SALES(I)/OFTQ(I) * FREQ
   GO TO 62
61 OFTQ(I) = SALES(I)/12. *.25
   OFTINV(I)  =  OFTQ(I)/2. * FREQ
   OFTORD(I)  =  SALES(I)/OFTQ(I) * FREQ
62 CONTINUE
   SIMINV  =  SIMINV + OFTINV(I)
   SIMORD  =  SIMORD + OFTORD(I)
63 CONTINUE
```

```
      CSTOFT  =  SIMORD * CSTORD + SIMINV * CSTINV
      DIFOFT  =  CSTMOC - CSTOFT
      PCTOFT  =  DIFOFT/DIFF * 100.
      WRITE (6,64)
   64 FORMAT (21X, 45H PLOT DATA FOR INVENTORY MODEL RESULTING FROM/
     116X, 56H ORDERING IN QUANTITIES INDICATED IN ORDER FACTOR TABLE./
     222X, 43H (NOTE - THIS IS NOT AN OPTIMAL INVENTORY)./)
      WRITE (6,65) SIMINV, SIMORD, CSTOFT, DIFOFT, PCTOFT
   65 FORMAT (19X, 40H WORKING INVENTORY LEVEL = ............$, F10.2/
     119X, 40H ANNUAL NUMBERS OF ORDERS CUT = ........, F8.0/
     219X, 40H TOTAL INVENTORY MANAGEMENT COST = ....$, F10.2/
     319X, 40H ANNUAL SAVINGS THROUGH USE OF O.F.T. =$, F10.2/
     419X, 40H PERCENTAGE OF MAX. POSSIBLE SAVINGS = ., F9.1//)
      GO TO 69
   66 WRITE (6,67)
   67 FORMAT (15X, 49H EOQ WORKING INVENTORY LEVEL DOES NOT EXCEED THAT/
     115X, 50H RESULTING FROM ORDERING ON A MONTHLY ORDER CYCLE./
     215X, 50H PROCEED TO ORDER IN QUANTITIES INDICATED IN ORDER/
     315X, 46H FACTOR TABLES OF NSSO STOCK CONTROL HANDBOOK.//)
      WRITE (6,68)
   68 FORMAT(16X, 48H * * * * * * * * * * * * * * * * * * * * * * * *//)
   69 CONTINUE
      END
5.0
      1.28        .10
 (50 cards of Sales data values - 649 items in all)
```

COMPUTER PRINT-OUT

DATA FOR 3245. SKU INVENTORY MODEL. ANNUAL
SALES DATA ON STAPLE STOCK ITEMS OBTAINED
FROM NAVY EXCHANGE, NAB LITTLE CREEK VA.
THIS MODEL IS OPERATING ON A MONTHLY ORDER BASIS

ITEM SALES	ITEM FREQ.	N	TOTAL ORDERS	ORDER QUANTITY	WORKING INVENTORY
$ 5.00	5	12	60	$ 0.42	$ 1.04
$ 15.00	5	12	60	$ 1.25	$ 3.12
$ 15.00	5	12	60	$ 1.25	$ 3.12
$ 17.00	5	12	60	$ 1.42	$ 3.54
$ 20.00	5	12	60	$ 1.67	$ 4.17
$ 7,125.00	5	12	60	$ 593.75	$ 1,484.37
$14,086.00	5	12	60	$1,173.83	$ 2,934.58
TOTALS	3245		38940		$119,937.44

PLOT DATA FOR 3245. SKU INVENTORY OPERATING
ON A MONTHLY ORDER CYCLE
ACTUAL SALES DATA OBTAINED FROM NAVEXCH LITTLE CREEK VA

WORKING INVENTORY LEVEL = ...$119,937.44
ANNUAL NUMBER OF ORDERS CUT = ... 38,940
TOTAL INVENTORY MANAGEMENT COST = ...$ 61,836.94
ORDER COST PER ITEM PER ORDER = ...$ 1.28
INVENTORY CARRYING COST - PERCENT PER ANNUM = .10

PLOT DATA FOR INVENTORY MODEL WITH ORDERING
PERFORMED ON AN ECONOMIC ORDER QUANTITY BASIS.
ACTUAL SALES DATA OBTAINED FROM NAVEXCH LITTLE CREEK.

WORKING INVENTORY LEVEL = ...$210,383.66
ANNUAL NUMBER OF ORDERS CUT = ... 16,436
TOTAL INVENTORY MANAGEMENT COST = ...$ 42,076.73
ANNUAL SAVINGS REALIZED THROUGH EOQ = $ 19,760.21
EOQ WORKING INVENTORY LEVEL HAS INCREASED TO 175.4
PERCENT OF LEVEL RESULTING FROM MONTHLY ORDER CYCLE

DATA FOR OPTIMAL POLICY CURVE PLOT INCLUDING
VALUES OF -K- FOR CALIBRATION OF O.P.C. CURVE.

WORKING INVENTORY	TOTAL ORDERS	K VALUE
$ 20790.36	166323	0.5
$ 41580.72	83161	1.0
$ 62371.08	55441	1.5
$ 83161.44	41581	2.0
$ 103951.79	33265	2.5
$ 124742.15	27720	3.0
$ 145532.51	23760	3.5
$ 166322.87	20790	4.0
$ 187113.23	18480	4.5
$ 207903.59	16632	5.0
$ 228693.95	15120	5.5
$ 249484.31	13860	6.0
$ 270274.66	12794	6.5
$ 291065.02	11880	7.0
$ 311855.38	11088	7.5
$ 332645.74	10395	8.0
$ 353436.10	9784	8.5
$ 374226.46	9240	9.0
$ 395016.82	8754	9.5
$ 415807.18	8316	10.0
$ 436597.54	7920	10.5
$ 457387.89	7560	11.0
$ 478178.25	7231	11.5
$ 498968.61	6930	12.0
$ 519748.97	6653	12.5
$ 540549.33	6397	13.0
$ 561339.69	6160	13.5
$ 582130.05	5940	14.0
$ 602920.41	5735	14.5
$ 623710.77	5544	15.0
$ 644501.12	5365	15.5
$ 665291.48	5198	16.0
$ 686081.84	5040	16.5
$ 706872.20	4892	17.0
$ 727662.55	4752	17.5
$ 748452.91	4620	18.0
$ 769243.27	4495	18.5
$ 790033.63	4377	19.0
$ 810823.99	4265	19.5
$ 831614.35	4158	20.0

ORDER QUANTITIES FOR EACH ITEM OF INVENTORY WHEN ORDERING PERFORMED ON OPTIMAL POLICY BASIS WITH INVENTORY RESTRAINT FIXED AT 100 PERCENT OF LEVEL CARRIED WHEN OPERATING ON MONTHLY ORDER CYCLE.

ANNUAL SALES PER ITEM	ORDER QUANTITY
$ 5.00	$ 6.45
$ 15.00	$ 11.17
$ 15.00	$ 11.17
$ 17.00	$ 11.89
$ 20.00	$ 12.90
$ 7125.00	$ 243.48
$ 14086.00	$ 342.34

PLOT DATA FOR OPTIMAL POLICY ORDER BASIS WITH RESTRAINT LIMITING WORKING INVENTORY TO 100. PERCENT OF LEVEL CARRIED WHEN OPERATING ON MONTHLY ORDER CYCLE. ACTUAL SALES DATA OBTAINED FROM NAVEXCH LITTLE CREEK VA

VALUE OF CONSTANT -K- FOR THIS INVENTORY = 2.884
WORKING INVENTORY LEVEL = ...$ 119937.45
ANNUAL NUMBER OF ORDERS CUT = ... 28831.
TOTAL INVENTORY MANAGEMENT COST = ...$ 48897.38
ANNUAL SAVINGS THROUGH OPTIMAL POLICY=$ 12939.56
PERCENTAGE OF MAX. POSSIBLE SAVINGS = . 65.5

PLOT DATA FOR OPTIMAL POLICY ORDER BASIS WITH RESTRAINT LIMITING WORKING INVENTORY TO 125. PERCENT OF LEVEL CARRIED WHEN OPERATING ON MONTHLY ORDER CYCLE. ACTUAL SALES DATA OBTAINED FROM NAVEXCH LITTLE CREEK VA

VALUE OF CONSTANT -K- FOR THIS INVENTORY = 3.606
WORKING INVENTORY LEVEL = ...$ 149921.82
ANNUAL NUMBER OF ORDERS CUT = ... 23065.
TOTAL INVENTORY MANAGEMENT COST = ...$ 44515.09
ANNUAL SAVINGS THROUGH OPTIMAL POLICY=$ 17321.85
PERCENTAGE OF MAX. POSSIBLE SAVINGS = . 87.7

PLOT DATA FOR OPTIMAL POLICY ORDER BASIS WITH RESTRAINT LIMITING WORKING INVENTORY TO 150. PERCENT OF LEVEL CARRIED WHEN OPERATING ON MONTHLY ORDER CYCLE. ACTUAL SALES DATA OBTAINED FROM NAVEXCH LITTLE CREEK VA

VALUE OF CONSTANT -K- FOR THIS INVENTORY = 4.327
WORKING INVENTORY LEVEL = ...$ 179906.16
ANNUAL NUMBER OF ORDERS CUT = ... 19221.
TOTAL INVENTORY MANAGEMENT COST = ...$ 42593.04

ANNUAL SAVINGS THROUGH OPTIMAL POLICY=\$ 19243.90
PERCENTAGE OF MAX. POSSIBLE SAVINGS = . 97.4

PLOT DATA FOR OPTIMAL POLICY ORDER BASIS WITH RESTRAINT LIMITING WORKING INVENTORY TO 175. PERCENT OF LEVEL CARRIED WHEN OPERATING ON MONTHLY ORDER CYCLE. ACTUAL SALES DATA OBTAINED FROM NAVEXCH LITTLE CREEK VA

VALUE OF CONSTANT -K- FOR THIS INVENTORY = 5.048
WORKING INVENTORY LEVEL = ...\$ 209890.53
ANNUAL NUMBER OF ORDERS CUT = ... 16475.
TOTAL INVENTORY MANAGEMENT COST = ...\$ 42076.85
ANNUAL SAVINGS THROUGH OPTIMAL POLICY=\$ 19760.10
PERCENTAGE OF MAX. POSSIBLE SAVINGS = . 100.0

PLOT DATA FOR INVENTORY MODEL RESULTING FROM ORDERING IN QUANTITIES INDICATED IN ORDER FACTOR TABLE. (NOTE - THIS IS NOT AN OPTIMAL INVENTORY).

WORKING INVENTORY LEVEL = ...\$ 211387.29
ANNUAL NUMBERS OF ORDERS CUT = ... 16614.
TOTAL INVENTORY MANAGEMENT COST = ...\$ 42404.65
ANNUAL SAVINGS THROUGH USE OF O.F.T. =\$ 19432.30
PERCENTAGE OF MAX. POSSIBLE SAVINGS = . 98.3

APPENDIX B

A MATHEMATICAL PROOF OF EQUATIONS III-6 AND III-7

<u>The Problem.</u> Prove the validity of the following statement:

To find the constant ($K_{R.I.}$) that will reduce the level of working inventory, in a given multi-item inventory, to a minimum while holding the number of orders processed annually at a constant number, divide the sum of the square roots of the item annual sales by the desired number of orders.[52]

$$\text{i.e.} \quad K_{R.I.} = \frac{\sum_i \sqrt{S_i}}{\sum_i N_i} \qquad \text{(Equation III-6)}$$

<u>Assumption.</u>

In a given inventory, the optimal relationship will exist between average working inventory and the total number of orders processed annually, when the individual order quantities are a function of the square root of the individual annual item sales, times a constant

<u>Solution.</u>

In any given multiple item inventory situation:

$$TVC = C_o \sum_i \frac{S_i}{q_i} + I \sum_i \frac{q_i}{2} \qquad (1)$$

Imposing a restraint on the total number of orders in the general form of:

$$\sum_i \frac{S_i}{q_i} = \sum_i N_i \qquad (2)$$

The problem is to minimize total variable costs of inventory management (TVC) subject to the given order numbers restriction. The mathematical technique for accomplishing this task is to use Lagrangian multipliers[53,54]. Writing the restraint as an equation equal to zero,

$$\sum_i \frac{S_i}{q_i} - \sum_i N_i = 0 \qquad (3)$$

We now form the Lagrangian (L) by adding λ, the multiplier, times equation (3) to (1), the equation to be minimized. This gives:

$$L = C_o \sum_i \frac{S_i}{q_i} + I \sum \frac{q_i}{2} + \lambda \left(\sum_i \frac{S_i}{q_i} - \sum_i N_i \right) = 0 \qquad (4)$$

We must then minimize L over q_i and L. Taking partial derivatives and equating them to zero we get:

$$\frac{\partial L}{\partial q_i} = - C_o \frac{S_i}{q_i^2} + \frac{I}{2} - \frac{\lambda S_i}{q_i^2} = 0 \qquad (5)$$

$$\frac{\partial L}{\partial \lambda} = \sum_i \frac{S_i}{q_i} - \sum_i N_i = 0 \qquad (6)$$

Solving for q_i in equation (5)

$$\frac{-\ 2C_oS_i \ +\ Iq_i^2 \ -\ 2\lambda S_i}{q_i^2} = 0$$

Then

$$q_i = \sqrt{\frac{2(C_o + \lambda)\ S_i}{I}} = \sqrt{\frac{2(C_o + \lambda)}{I}} + \sqrt{S_i} \tag{7}$$

Substituting the value of q_i from (7) into (6) we get:

$$\sum_i \frac{\sqrt{S_i}}{\sqrt{\frac{2(C_o+\lambda)}{I}}} - \sum_i N_i = 0 \tag{8}$$

and

$$\sqrt{\frac{2(C_o+\lambda)}{I}} = \frac{\sum_i \sqrt{S_i}}{\sum_i N_i} \tag{9}$$

where

$$\sqrt{\frac{2(C_o+\lambda)}{I}} = K_{R.I.} = \frac{\sum_i \sqrt{S_i}}{\sum_i N_i} \tag{10}$$

Q.E.D.

The proof of equation III-7 can be obtained in the same manner.

<u>The problem.</u>

To find the constant ($K_{R.O.}$) that will reduce the number of orders in a given inventory to a minimum while holding

the working inventory level constant, divide the sum of the order quantities by the sum of the square roots of the annual sales.

$$\text{i.e.} \qquad K_{R.O.} = \frac{\sum_i Q_i}{\sum_i \sqrt{S_i}} \qquad \text{Equation III-7}$$

<u>Assumption.</u> Same as above.

<u>Solution.</u>

The same rationale is applied to the proof of Eq. III-7 as to Eq. III-6.

$$TVC = C_o \sum_i \frac{S_i}{q_i} + I \sum_i \frac{q_i}{2} \qquad (1A)$$

Imposing a restraint on working inventory level in the form of:

$$\sum_i \frac{q_i}{2} = \sum_i INV_i \qquad (2A)$$

and proceeding as before.

$$\sum_i \frac{q_i}{2} - \sum_i INV_i = 0 \qquad (3A)$$

$$L = C_o \sum_i \frac{S_i}{q_i} + I \sum \frac{q_i}{2} + \lambda \left(\sum_i \frac{q_i}{2} - \sum_i INV_i \right) = 0$$

(4A)

$$\frac{\partial L}{\partial q_i} = -\frac{C_o S_i}{q_i^2} + \frac{I}{2} + \frac{\lambda}{2} = 0 \qquad (5A)$$

$$\frac{\partial L}{\partial \lambda} = \sum_i \frac{q_i}{2} - \sum_i INV_i = 0 \qquad (6A)$$

$$\frac{-2C_o S_i + Iq_i^2 + \lambda q_i^2}{2q_i^2} = 0$$

$$q_i = \sqrt{\frac{2C_o S_i}{(I+\lambda)}} = \sqrt{\frac{2C_o}{(I+\lambda)}} \times \sqrt{S_i} \qquad (7A)$$

Substituting the value of q_i from (7A) into (6A)

$$\frac{\sqrt{\frac{2\ C_o}{(I+\lambda)}} \times \sum_i \sqrt{S_i}}{2} - \sum_i INV_i = 0 \qquad (8A)$$

and

$$\sqrt{\frac{2\ C_o}{(I+\lambda)}} = \frac{2 \sum_i INV_i}{\sum_i \sqrt{S_i}} \qquad (9A)$$

where

$$\sqrt{\frac{2\ C_o}{(I+\lambda)}} = K_{R.O.} \qquad (10A)$$

and $2 \sum_i INV_i = \sum_i Q_i$ at optimal conditions.

Therefore

$$K_{R.O.} = \frac{\sum_i Q_i}{\sum_i \sqrt{S_i}} \qquad (11A)$$

Q.E.D.

Note. In the proofs offered above, the Lagrangian multiplier λ is not an arbitrary mechanical device. It plays the role of a cost in formulation of the Lagrangians (L). In each case λ is intimately related to the imputed costs resulting from pursuing optimal order policies under restrictions. In most economic problems in which it is necessary to use Lagrangian multiplier methods, it develops similarly that λ has a very basic economic significance.

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